Proceeding of the IEEE
International Conference on Robotics and Biomimetics (ROBIO)
Shenzhen, China, December 2013

Grasp Planning using Quadric Surface Approximation for Parallel Grippers

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***Abstract*— We propose a planning method for a gripper grasping daily life objects using quadric surface approximation. This method can decompose the objects to the approximate quadric surfaces. The planner can find regions for contacting fingers quickly so that the gripper grasps the object firmly. The planner evaluates the grasp stability for these postures. The previous evaluation method in surface contact considers contact area only. We extend the evaluation method for considering stress distribution. We performed simulation and verified the effectiveness of our grasp planning.**

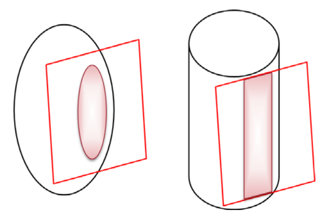

Fig. 1. Differences of the shape of contact surface

I. INTRODUCTION

Service robots are expected to grasp daily life objects which have variety of shapes. The robots have to find hand positions, postures and contact points which satisfy the condition of grasp stability in the large configuration space of them. Several methods are proposed for finding the positions efficiently using shape decomposition of the object. In these methods, the object is decomposed to several pieces such as shape primitive and approximated surfaces. The hand postures are given by adding offset to each piece and grasp styles are generated so that fingers pinch or wrap the piece.

Goldfeder et al.[5] proposed a method that approximate the object by the combination of several super-quadric surfaces. This method decomposes the object so that each piece fits a super-quadric surface. The approximated surface is convex, closed and point symmetry. It is suitable for detecting convex pieces of the whole object. However, the super-quadric surface may not fit some surfaces, since the object surface often includes concave surfaces and asymmetry surfaces.

In this paper, we propose a method for approximating objects by the combination of several quadric surface regions. Since arbitrary curved surface regions are fitted by the quadric approximation equation which can express local features. Even though the object surface includes concave region, the object surface is decomposed appropriately. The grasp posture for parallel gripper with soft material generates quickly using the quadric parameter.

This work was supported by JSPS KAKENHI Grant Number 24700194.

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In addition, we propose a method that evaluates grasp stability for surface contact take account of distribution of stress. This method estimate the curvature of contact region using the proposed quadric surface approximation. The curvature affects to the shape of the contact region and a distribution of stress which is applied by the finger to the surface. Examples of the shape of contact region when grasping an ellipsoid and an elliptic surface are shown in **Fig.1**. The moment in the direction of contact region normal is calculated using the stress distribution. Ciocarlie et al.[16] proposed the evaluation method which considers only the area of the contact region. We extend this method for considering the stress distribution. This evaluation is effective for finding better quality grasp than the previous methods.

We simulate robot grasping of daily life objects. The effectiveness of our method is conformed in the simulation.

II. RELATED WORKS

Grasp planning methods[1][2] based on random search have been proposed. For reducing the searching time, several methods using approximating a target object are proposed. For example, Yamanobe and Nagata[4] proposed a method that uses various primitive shapes such as a sphere, a cylinder, a cone, a box and a tubular box, a tubular cylinder. In this method, users need to assign primitives to the object by hand. Goldfeder et al.[5] proposed the method that approximate the object by the combination several super-quadric surfaces. A super-quadric surface can express more variety of shapes than a quadric surface. However this method limit the super-quadric surface as only convex super-ellipsoids. Therefor opened surfaces and concave surfaces are not able to being expressed. Harada et al.[6] proposed the method of the grasp planning using a gripper. The object is decomposed to coplanar surfaces and two parallel surfaces of them are selected as finger contact regions. Curved surfaces are not always decomposed appropriately for finding parallel surfaces.

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As the technique of evaluation the grasp, the method is proposed based on force closure[7][8][9], and it is extended a lot, previously. Force Closure was the concept that is originally introduced to kinematics[7] and was introduced to field of the robot hand[9]. Nguyen[10] discussed about the constitution method of Force Closure for robot hand, and his approach is extended by many researches. For example, Ferrari and Canny[11] proposed the metric for grasp stability evaluation using grasp wrench space. D. J. Montana[12][13] proposed the grasp stability evaluation considering the curvatures of both the hand and the object. Rimon and Burdick[14] proposed the mobility index considering curvatures at contact points. Funahashi et al.[15] also analyze grasp stability considering curvatures at contact points. These methods are the case of the contact point with the rigid bodies and fingers; therefore, they don't consider the moment in the normal direction at contact surface. Ciocarlie et al.[16] proposed the metric for surface contact considering the moment in the normal direction of the surfaces. However, this method only considers areas of contact regions. The curvature of the surfaces are used for only calculating contact regions. In this paper,we extend this method to a method that analyzes the stability considering a contact region shape and the stress distribution.

III. Overview of grasp planning

The planner approximates a target object by the combination of quadric surfaces. Ellipsoids and elliptic cylinders are extracted from them. The surface which has curvatures larger than a threshold is not selected, since the surface is not suitable for grasp. Grasp postures are generated for the selected quadric surfaces. If two grippers contact the target quadric surface, grasp stability is evaluated. After all of generated postures is checked, several more stable postures are extracted.

IV. Quadric surface approximation

In this section, we describe the quadric approximation. The fitting error of quadric surface is calculated using a least square method. The object surface clustering is performed using this error. The generated clusters are classified by quadric parameters.

A. Approximation of mesh models

We assume that the polygon model is given for each object. Let $f(x, y, z) = 0$ be an implicit function of a quadric surface defined as:

$$\begin{aligned} f(x,y,z) &= \boldsymbol{a}\cdot\boldsymbol{p} \qquad (1)\\ &where\\ \boldsymbol{a} &= [a_0\ a_1\ a_2\ a_3\ a_4\ a_5\ a_6\ a_7\ a_8\ a_9]^T\\ \boldsymbol{p} &= [x^2\ y^2\ z^2\ xy\ yz\ zx\ x\ y\ z\ 1]^T \end{aligned}$$

We have to acquire the parameter $\boldsymbol{a}$ to minimize the distance between the quadric surface $f(x, y, z) = 0$ and original object surface. The Taubin[17] method for least squared fitting is used as the following equation:

$$\boldsymbol{M}\boldsymbol{a} = \lambda\boldsymbol{V}\boldsymbol{a} \qquad (2)$$

$where$

$$\begin{aligned} \boldsymbol{M} &= \Sigma_i \boldsymbol{p}_i\boldsymbol{p}_i^T\\ \boldsymbol{V} &= \Sigma_i \Delta\boldsymbol{p}_i\Delta\boldsymbol{p}_i^T\\ \boldsymbol{p}_i &= [x_i^2\ y_i^2\ z_i^2\ x_iy_i\ y_iz_i\ z_ix_i\ x_i\ y_i\ z_i\ 1]^T\\ \Delta\boldsymbol{p}_i &= \begin{bmatrix} 2x_i & 0 & 0 & y_i & 0 & z_i & 1 & 0 & 0 & 0\\ 0 & 2y_i & 0 & x_i & z_i & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 2z_i & 0 & y_i & x_i & 0 & 0 & 1 & 0 \end{bmatrix}^T \end{aligned}$$

where $\boldsymbol{M}$ is a covariance matrix and $\boldsymbol{V}$ is constraint matrix and (x_i, y_i, z_i) denotes the i-th vertex of the polygon model. We solve general eigen problem for the eigenvector $\boldsymbol{a}$ which corresponds to the minimum eigen value λ. The $\boldsymbol{a}$ minimizes $\boldsymbol{a}^T\boldsymbol{M}\boldsymbol{a}$ in the constraint $\boldsymbol{a}^T\boldsymbol{V}\boldsymbol{a} = 1$.

Here, it is desirable to use the surface integral instead of Eq.(2) for being robust against the difference of density of data points. By using the area of the triangular patch of polygon model, $\boldsymbol{M}$ and $\boldsymbol{V}$ can be calculated by using the surface integral as follows:

$$\boldsymbol{M} = \int_r \boldsymbol{p}\boldsymbol{p}^T dA \qquad (3)$$

$$\boldsymbol{V} = \int_r \Delta\boldsymbol{p}\Delta\boldsymbol{p}^T dA \qquad (4)$$

where r is surface region and dA is area element.

Eq.(2) is not robust for numerical error, since $\boldsymbol{V}$ is singular matrix. Then we transform this equation to non-singular matrix using the same approach of Halir[18]. We decompose the $\boldsymbol{M}$, $\boldsymbol{V}$ and $\boldsymbol{a}$ as bellows.

$$\boldsymbol{M} = \left(\begin{array}{c|c} \tilde{\boldsymbol{M}} & \boldsymbol{m}_2\\ \hline \boldsymbol{m}_2^T & m_3 \end{array}\right) \qquad (5)$$

$$\boldsymbol{V} = \left(\begin{array}{c|c} \tilde{\boldsymbol{V}} & \boldsymbol{0}\\ \hline \boldsymbol{0}^T & 0 \end{array}\right) \qquad (6)$$

$$\boldsymbol{a} = \left[\begin{array}{c} \tilde{\boldsymbol{a}}\\ \hline a_9 \end{array}\right] \qquad (7)$$

Eq.(2) is replaced as below:

$$\tilde{\boldsymbol{M}}\tilde{\boldsymbol{a}} + \boldsymbol{m}_2 a_9 = \lambda\tilde{\boldsymbol{V}}\tilde{\boldsymbol{a}} \qquad (8)$$

$$\boldsymbol{m}_2^T\tilde{\boldsymbol{a}} + m_3 a_9 = 0 \qquad (9)$$

Then a_9 of Eq.(9) is substituted in Eq.(8):

$$\left(\tilde{\boldsymbol{M}} - \frac{\boldsymbol{m}_2\boldsymbol{m}_2^T}{m_3}\right)\tilde{\boldsymbol{a}} = \lambda\tilde{\boldsymbol{V}}\tilde{\boldsymbol{a}} \qquad (10)$$

After the part of coefficient vector $\tilde{\boldsymbol{a}}$ is calculated as an eigen vector, then $\tilde{\boldsymbol{a}}$ is substituted to Eq.(9) and a_9 is acquired.

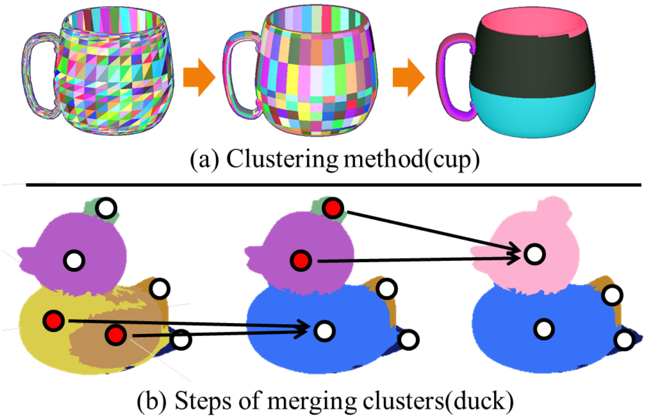


Fig. 2. Models used for simulation

TABLE I
CLASSIFICATION OF QUADRIC SURFACES

type	condition
Ellipsoid	$u > 0, v > 0, w > 0$
Cylinder	$u > 0, v > 0, w \simeq 0$
	$u > 0, v \simeq 0, w > 0$
	$u \simeq 0, v > 0, w > 0$

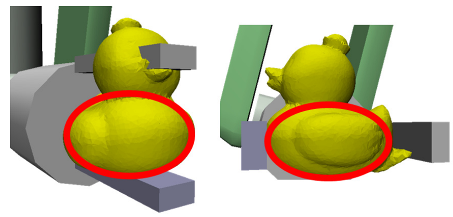


Fig. 3. Examples of exceptions for grasp postures
(Red ellipses are target surface)

This method needs exception processing for planes since some objects have plane regions. If the surface is planar, the $\tilde{\boldsymbol{V}}$ is the singular matrix. Then, the surface is regarded as a plane, if the approximation error for the plane is smaller than a threshold.

The other problem of quadric approximation is that the $\boldsymbol{a}$ can be two planes. The result is not preferred for this application, since the two different planes are regarded as one approximated surface area. Then such solutions of Eq.(8) are eliminated.

The calculated λ is the fitting error of the quadric equation. It is used for generating quadric surface clusters as described in the next section.

B. Quadric surface approximation

Michael et al.[19] proposed the method for generating face clusters which are connected sets of triangles. By using the same procedure of the clustering method, the hierarchy structure that has quadric surface node is generated. In the initial state, all clusters including a triangle in the mesh models. Adjacent face clusters are merged in ascending order of the approximation error. The fitting error of all adjacent pairs are calculated and one pair which has the least error is selected as the new generated cluster. This procedure are repeated iteratively and the cluster grows. If approximation error exceeds the threshold, merging procedure is finished. Examples of the merging process are shown in **Fig.2**.

C. Selecting quadric surface

Eq.(1) is transformed as the general form of implicit equation:

$$ux'^2 + vy'^2 + wz'^2 = 1 \tag{11}$$

In this equation, the parameters u, v and w determine a type of the quadric surface. **Table.I** shows the relation of the surface types and conditions of u, v and w.

V. GENERATION OF GRASP POSTURES

In this section, a process of generation grasp posture is described. We use the two-finger parallel gripper made of soft material. Several candidates of postures for an ellipsoid and an elliptic cylinder are generated, and the grasp stability are evaluated. The grasp stability for each candidate is evaluated using a method described in **Section.VI**. The following posture is excepted evaluation, and examples are shown in **Fig.3**.

- There is a collision between a finger and the surface which is not target quadric surfaces
- Target objects are sharper than a threshold

A. Ellipsoid

Quadric surfaces whose parameters u, v and w are positive are selected from quadric surfaces. For selected ellipsoids, three principal axes $\boldsymbol{r_1}$, $\boldsymbol{r_2}$ and $\boldsymbol{r_3}$ are calculated. The one axis is chosen as the hand approaching axis from these and the one axis is chosen as the gripper closing axis from remaining two axes, as shown in **Fig.4(a)**. The approaching axis has a variation of 6 directions which are positive and negative directions of principal axes. The gripper closing axis has variation of 4 directions as rotation by 90 degrees. Then, a ellipsoid has 24 candidates of grasp postures.

B. Elliptic cylinder

The system selects that parameters u, v and w satisfy the conditions as cylinder from quadric surfaces. For selected the elliptic cylinder, two principal axes $\boldsymbol{r_1}$ and $\boldsymbol{r_2}$ of ellipses of the end faces are calculated. The one axis is chosen as the approaching axis from these and the other one is as the gripper closing axis. The approaching axis has variation of 4 directions which are positive and negative directions of the two axes. The gripper closing axis has variation of 2 directions as rotation by 180 degrees. As shown Fig.4(b), upper point, middle point and lower point of the cylinder are set as grasping points. Let l be a length between the end faces of the cylinder. The upper point and the lower point are defined as the position that is moved to $+l/4$ and $-l/4$ from middle point respectively. Then, the elliptic cylinder has 24 candidates of grasp postures.

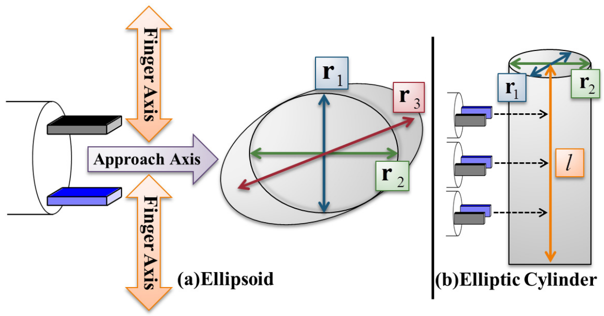


Fig. 4. Grasp postures for ellipsoid and elliptic cylinder

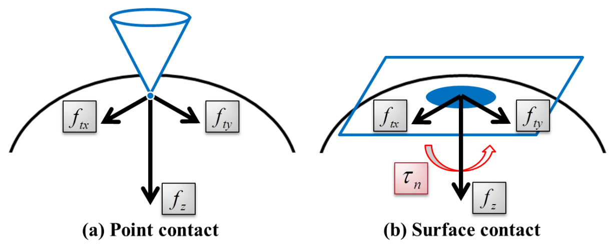


Fig. 5. Contact stress distribution

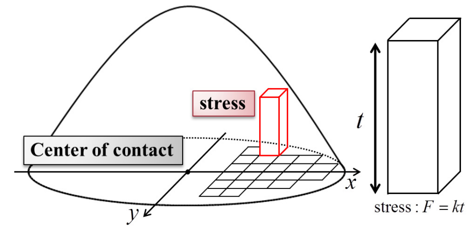


Fig. 6. Contact stress distribution

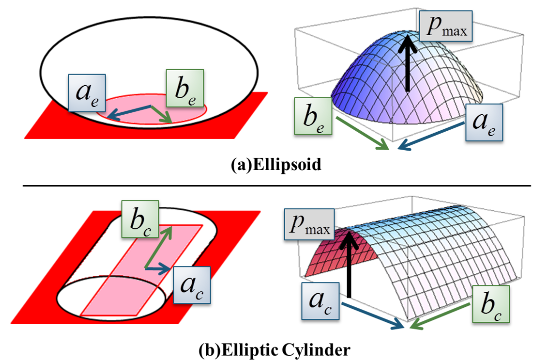


Fig. 7. Stress models based on Winkler elastic foundation

VI. Evaluation of the Grasp Stability

The evaluation method of the grasp stability is categorized in two: point contact and surface contact. The point contact is realized using a hand whose finger-tip is made of a rigid material. The evaluation methods in the point contact[10]-[15] were proposed previously. These methods don't need considering the moment in the normal direction at contact point so that the moment isn't developed as shown **Fig.5**(a).

We use the gripper whose surface is a plane and is pasted a soft material; therefore, our method assumes surface contact between grasp object surface and plane. In the case that the grasp stability in surface contact is evaluated, it is needed considering the moment in the normal direction at contact surface as shown Fig.5(b). As the evaluation method in surface contact, Ciocarlie et al.[16] proposed the metric for surface contact. We extend this method for more accurate evaluation of grasp stability.

A. Evaluation Based on Force Closure

Previous method[16] evaluates the grasp stability in considering of the moment which acts around the normal direction of contact plane between convex shapes. The frictional condition is shown the following form:

$$f_t{}^2 + \frac{\tau_n{}^2}{e_n{}^2} \leq \mu^2 p^2 \qquad (12)$$

where f_t is magnitude of the tangent plane of contact, p is magnitude of a total load and μ is the frictional coefficient, τ_n is magnitude of a frictional moment. e_n is referred as the eccentricity parameter, and it is shown the following equation:

$$e_n = \frac{max(\tau_n)}{max(f_t)} \qquad (13)$$

B. Calculation of the maximum static frictional torque

We describe about the calculation method of the maximum static frictional torque $max(\tau_n)$ in Eq.(13). **Fig.6** shows the distributional pattern diagram of contact stress. The contact surface is divided to the micro region. The static frictional moment $\Delta\tau(x,y)$ which generates at the micro region is expressed the following:

$$\Delta\tau(x,y) = \sqrt{x^2+y^2}\mu s(x,y)$$

where $s(x,y)$ is the stress generating at unit area of the micro region and changes its distribution by shapes of grasped objects. By integrating $\Delta\tau(x,y)$, the maximum static frictional torque $max(\tau_n)$ is expressed:

$$max(\tau_n) = \int\int_D \Delta\tau(x,y)dxdy \qquad (14)$$

As shown in Eq.(14), when the large stress generates far from the center of the contact surface, $\Delta\tau(x,y)$ becomes large.

C. Modeling of the stress distribution of contact region

Winkler elastic foundation[20] is stress model that is approximated stress distribution by quadratic in elastic contact. We extend previous method[16] to the evaluation method. In this following, stress distributions and stress models for each of surface are described where p_{max} is the maximum of stress.

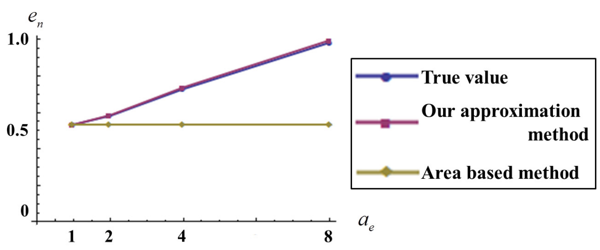


Fig. 8. Comparison with the previous method

1) Ellipsoid: In the case of grasping an ellipsoid, the stress model is shown in Fig.7(a), and the stress distribute $s(x, y)$ is shown the following equation:

$$s(x, y) = p_{max}\left(1 - \left(\frac{x}{a_e}\right)^2 - \left(\frac{y}{b_e}\right)^2\right) \quad (15)$$

where a_e, b_e is a longer axis and a shorter axis of elliptic contact surface. e_n is derived by Eq.(15) and shown the following equation:

$$e_n = \frac{16}{15}\frac{a_e}{\pi}E\left[\frac{\pi}{2}, 1 - \left(\frac{b_e}{a_e}\right)^2\right] \quad (16)$$

This equation can be approximated the following equation:

$$e_n = \frac{8}{15\pi}\sqrt{4(a_e - b_e)^2 + \pi^2 a_e b_e} \quad (17)$$

On the other hand, [16] method derived e_n as the following equation:

$$e_n = \frac{8}{15}\sqrt{a_e b_e}$$

This method considers only the case that the shape of contact regions is circle in which a_e equals b_e . In our approximation method, e_n is able to be derived in case that the shape of contact surface is an ellipse. The difference between e_n of our method and the one of [16] method is shown in **Fig.8** when $a_e b_e$ fixes 1 and the ratio between a_e and b_e is changed, for example $(a_e, b_e) = \{(1, 1), (2, 1/2), (4, 1/4), (8, 1/8)\}$. If a gripper and a object contact at a edge, the grasp stability can be calculated using our method.

2) Elliptic Cylinder: In the case of grasping an elliptic cylinder, the stress model is shown in Fig.7(b), and the stress distribute $s(x, y)$ is shown the following equation:

$$s(x, y) = p_{max}\left(1 - \left(\frac{x}{a_c}\right)^2\right) \quad (18)$$

where a_c is the one of side in the contact surface parallel to generatrix of cylinder, and where b_e is the other hand. e_n is derived by Eq.(18) and shown the following equation:

$$e_n = \frac{1}{80a_c^3 b_c}\Bigg(a_c b_c(22a_c^2 - 3b_c^2)\sqrt{a_c^2 + b_c^2} + 8a_c^2 \log\frac{b_c + \sqrt{a_c^2 + b_c^2}}{a_c} + b_c^3(20a_c^2 + 3b_c^2)\log\frac{a_c + \sqrt{a_c^2 + b_c^2}}{b_c}\Bigg) \quad (19)$$

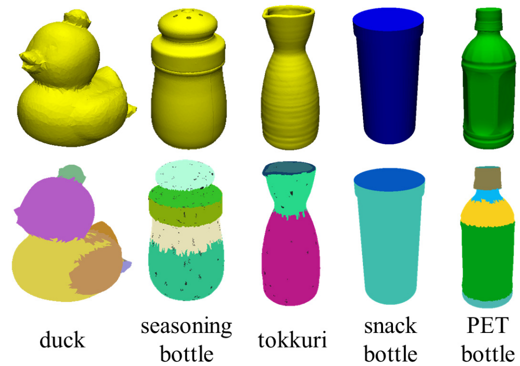


Fig. 9. Models used for simulation

VII. Simulation

We simulate grasp planning using five models, a duck, a seasoning bottle, a sake bottle, a snack bottle and a PET bottle. Each of models is shown in **Fig.9**, where the top of Fig.9 shows mesh models, and the bottom of Fig.9 shows the model applied color coding each of quadric surfaces.

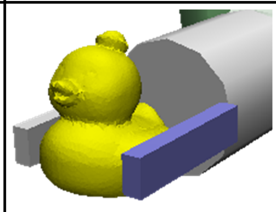
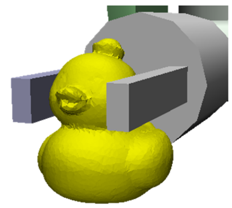
Candidates of grasp posture are planned and the grasp stability is evaluated. Grippers move 2mm toward an object from a surface of the target object. The target quadric surface, the grasp posture, the contact area and the value of evaluation is shown in **Fig.10**. This value of evaluation points that is described by Ferrari and Canny[11].

For a duck model, we plan grasping a body ellipsoid that the volume is the largest and a head ellipsoid that the volume is the second largest. In addition, for the body ellipsoid, the result of simulation that hand approaches from difference direction is shown. For the seasoning bottle and the sake bottle, we plan grasping the biggest ellipsoid of the approximated surfaces.

Previous evaluation method[16] use the convex finger and the convex object; therefore, this method is able to establish a stress distribution and a shape of contact surface without depending target surface shape, uniquely. Although, if the plane gripper is used for grasping the convex object, the stress distribution and the shape of contact surface changes depending on target surface shape. Additionally, the grasp stability depends on only an area of the contact surface between the gripper and the target surface in previous method. In our method using the plane gripper, we can confirm that the grasp stability depends on the stress distribution.

VIII. Conclusion

In this paper, we have proposed a grasp planning using the plane gripper. The planner approximates a target object by the combination of quadric surfaces which fit local features. Ellipsoids and elliptic cylinders are selected from them, and several grasp postures are generated. We have proposed the evaluation method take account of stress distribution and the shape of contact surface. We show that our planner generate good quality grasp by simulation.

Shape	Ellipsoid(body)	Ellipsoid(body)	Ellipsoid(head)
			
Area[cm^2]	4.05	2.97	4.10
Evaluation	0.0321	0.0280	0.0322

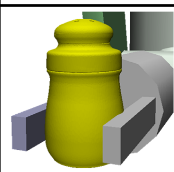
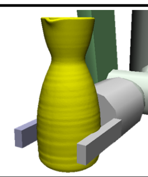
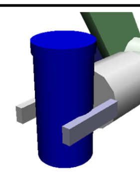
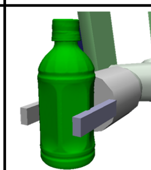
Shape	Ellipsoid	Ellipsoid	Elliptic cylinder	Elliptic cylinder
				
Area[cm^2]	3.94	4.70	4.34	2.57
Evaluation	0.0316	0.0342	0.0418	0.0323

Fig. 10. Result of simulation

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