2015 IEEE International Conference on Advanced Intelligent Mechatronics (AIM)
July 7-11, 2015. Busan, Korea

Altitude Estimation Using Particle Filter with Monopulse Radars in a Multipath Environment

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Abstract— **The problem of multipath propagation in the tracking low-altitude targets with a radar is addressed. It is well known that multipath fading causes bias errors to the target altitude over the sea. Since the bias error caused by multipath propagation depends on a large number of parameters such as the frequency of the radar waveform, the actual target altitude, and range, it is difficult to estimate the bias errors. In this paper, we propose an altitude estimation method using particle filter and multipath propagation model. The performance of the proposed method is verified through computer simulations.**

I. INTRODUCTION

It is well known that the problem of multipath propagation (MP) arises in the situation of tracking low-altitude targets with a radar and causes large bias errors to the real altitude of the target. In the presence of multipath propagation, interference among the direct path and surface-reflected ones is caused. The interference affects bias errors of the target angle. Estimation of the bias errors is normally a difficult problem due to the dependence on a large number of parameters such as the frequency of the radar waveform, radar altitude above the sea, the actual target altitude, and the range from a radar to the target and the surface reflectivity[1].

Tracking algorithm for MP problem have been widely studied[2]-[7]. A conventional tracking method for MP problem is able to eliminate large peak errors (spike errors) but is not able to eliminate constant bias errors to the real altitude of the target over the sea. On the other hand, we proposed altitude estimation algorithm which calculates the reliability of assumed altitude hypothesis, which is chosen from several preset altitude hypothesis, using bias error based on multipath propagation model[8]. However, there is a problem that the altitude estimation method mentioned above is able to estimate only the preset altitude. Therefore, the accuracy of the altitude estimation method should be degraded when the altitude except the preset ones is estimated.

In this paper, we propose an altitude estimation method using a particle filter based on MP model. Our method can compensate the constant bias errors to the real altitude of the target through tracking. Compared with conventional tracking method which regards bias errors as random errors, proposed method calculates the importance weight of assumed altitudes, which corresponds to particles in particle filter, incorporating bias error based on multipath propagation model to estimate target altitude. Moreover, compared with our conventional method as stated above, our proposed method is able to estimate the altitude but the preset ones by using particle filter theorem. Therefore, the accuracy of altitude estimation is much improved by the proposed method.

This paper is organized as follows. MP model is described in Section Ⅱ. Proposed method is described in Section Ⅲ. Simulation results for 3 simulation scenarios are presented in Section Ⅳ. Finally, a summary of the results is given in Section Ⅴ.

II. MULTIPATH PROPAGATION MODEL

The geometry of multipath phenomenon is shown as Fig.1. There are two separate paths between the target and the radar: the direct path and the indirect path via reflection with grazing angle ψ_g from the sea (or ground) surface.

The surface reflected signal consists of two components, that is, specular and diffuse. The specular reflection is caused by a smooth (mirror-like) surface and the diffuse reflection is caused by the surface irregularities. While the specular reflection coefficient is a deterministic number which depends on several parameters, the diffuse reflection has a random nature. Therefore, in this paper, we do not take into account the diffuse reflection because a proposed tracking method using a particle filter is able to remove random errors by the diffuse reflection.

Under precondition as above, signal-to-noise ratio (SNR) and the constant bias errors to the target altitude is depended on specular reflection at the surface of indirect path and relative phase between direct and indirect paths[1].

In this Section, we present the mathematical expressions for the elevation angle error, SNR, and the probability of detection by specular reflection.

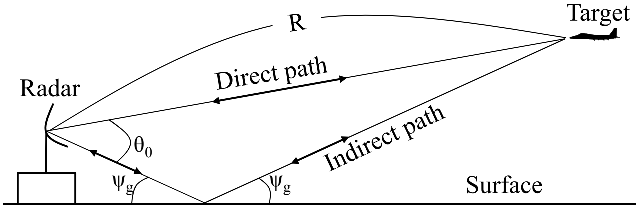

Figure 1. Geometry of multipath propagation.

A. *The elevation angle error*

The elevation angle error with respect to the target is given by[7]

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$$\Delta E = -\frac{\rho(\rho + \cos\Delta\Phi)}{1 + 2\rho\cos\Delta\Phi + \rho^2}\theta_0 \qquad (1)$$

where ρ is the specular reflection coefficient, $\Delta\Phi$ is the relative phase between the direct path and the indirect path and θ_0 is the angle difference between the two paths in Fig.1. The specular reflection coefficient ρ in (1) is composed of three factors: Fresnel reflection coefficient Γ, specular scattering factor ρ_s and divergence factor D and is given by[7]

$$\rho = |\Gamma| \rho_s D \qquad (2)$$

The Fresnel reflection coefficient for a smooth surface is determined by the electromagnetic properties of the surface. The Fresnel reflection coefficient for vertical polarization is given by[7][9]

$$\Gamma = \frac{\varepsilon_c \sin\psi_g - \sqrt{\varepsilon_c - \cos^2\psi_g}}{\varepsilon_c \sin\psi_g + \sqrt{\varepsilon_c - \cos^2\psi_g}} \qquad (3)$$

In the above equations, ψ_g is the grazing angle in Fig.1 and ε_c is the complex dielectric constant, given by[9]

$$\varepsilon_c = \varepsilon_r - j\varepsilon_i \qquad (4)$$

$$\varepsilon_r = \frac{\varepsilon_s - \varepsilon_0}{1 + (2\pi f \tau)^2} + \varepsilon_0 \qquad (5)$$

$$\varepsilon_i = \frac{2\pi f \tau(\varepsilon_s - \varepsilon_0)}{1 + (2\pi f \tau)^2} + \frac{2\sigma_i}{f} \qquad (6)$$

where f is radar frequency, ε_0 is constant(ε_0 =4.9), ε_s is static dielectric constant, τ is attenuation coefficient, and σ_i is ion conductivity. In the following simulation, ε_s, τ and σ_i is set to be 69.1, 9.2×10^{-12}, 4.7×10^{-10}, respectively[9].

The specular scattering factor ρ_s in (2) is given by [7]

$$\rho_s = \exp\left[-2\left(\frac{2\pi\sigma_h \sin\psi_g}{\lambda}\right)^2\right] \qquad (7)$$

where the grazing angle is the angle between indirect path and surface which depends on target altitude, and λ is the wavelength of the waveform. σ_h is the rms(root-mean-square) of the waveheight.

Divergence factor is another quantity that affects the reflection coefficient. This factor is considered due to the curvature of the Earth. The reflected waves from a surface diverge and this divergence causes an attenuation in the power density of the waves. An approximate value for the divergence factor is given by[1][7]

$$D = \left(1 + \frac{2r_1 r_2}{r_e r \sin\psi_g}\right)^{-\frac{1}{2}} \qquad (8)$$

where r_1 and r_2 are the ground distances from the reflection point to the radar normal projection and the target normal projection points, respectively, $r = r_1 + r_2$,and r_e is Earth radius.

Fig.2 shows an example of the measured target altitude including ΔE . The horizontal axis is the range from the radar to the target, the vertical axis is the target altitude, solid line represents the true altitude and solid line with cross sign represents the measured target altitude. As can be seen from Fig.2, the elevation angle error ΔE in (1) depends on the target position.

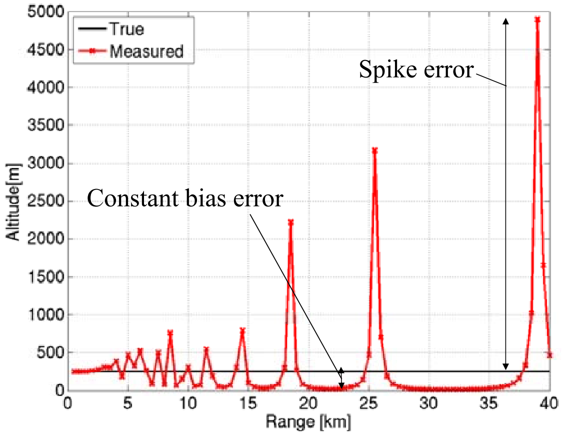

Figure 2. True and measured altitude.

B. *SNR(signal-to-noise ratio)*

The SNR on MP model is given by[1]

$$SNR_{mlt} = SNR_0 + 40\log_{10}\left(F \cdot \frac{R_{d0}}{R}\right) \qquad (9)$$

where SNR_0 is SNR at the reference range R_{d0} , R is the range from the radar to the target and F is the pattern propagation factor. In addition, F is given by[1][7]

$$F = \left|1 + \rho e^{j\Delta\Phi}\right| = \sqrt{1 + 2\rho\cos\Delta\Phi + \rho^2} \qquad (10)$$

C. *Probability of Detection*

In this paper, assuming that the target fluctuates according to a Swerling case I[10], detection probability *Pd* is given by

$$Pd = f_d(R) = \exp\left(\frac{\log(P_{fa})}{(1+G)}\right) \qquad (11)$$

where P_{fa} is false-alarm probability and G is antilogarithm of SNR in (9).

III. PROPOSED METHOD

MP errors depend on the real altitude of the target and the range from the radar to the target which are unknown. Therefore, we propose a method which substitutes assumed altitudes for the actual altitude utilizing particle filter, where the assumed altitude is regarded as particle, to estimate the target altitude. Our proposed method calculates the importance weight w_m of an assumed altitude $z_{a,m}$ which is calculated by using estimated horizontal state vector, the altitude $h_{mlt,m}$ including the constant bias errors based on the MP model(where $h_{mlt,m}$ is called "multipath altitude" in this paper, in addition, the subscript "m" means the model number from 1 to M which identify an assumed altitude, M means the maximum number of assumed altitudes and the subscript "a" means the assumed value). And then, proposed method estimates the altitude of the target using assumed altitudes and the importance weight of those. Fig.3 shows the block diagram of our proposed method. As shown in Fig.3, our proposed method is composed of 6 blocks (which are "Initialization", "Tracking", "Multipath propagation generator", "Importance calculator" , "Resampling" and "Altitude Estimator"). The processes of our method are shown as the following.

a) Initialization

"Initialization" process sets the assumed altitudes in particles within a predetermined range at equal intervals (See table Ⅱ).

b) Tracking (Section A)

"Tracking" process calculates target state estimates and their covariance matrix using tracking filter. This process also outputs correlation result (e.g., radar measurements are in/out of tracking gate).

c) Multipath propagation generator (Section B)

"Multipath propagation generator" calculates the multipath altitude $h_{mlt,m}$ for each assumed altitude $z_{a,m}$(which is set in advance) using track estimates x_s, y_s, and radar frequency f.

$\boldsymbol{z}_a$ means assumed altitude set, $\boldsymbol{h}_{mlt}$ means multipath altitude set as follows.

$$\boldsymbol{z}_a = \left(z_{a,1}, z_{a,2}, \cdots, z_{a,M}\right)$$

$$\boldsymbol{h}_{mlt} = \left(h_{mlt,1}, h_{mlt,2}, \cdots, h_{mlt,M}\right)$$

d) Importance weight calculator (Section C)

"Importance weight calculator" caluculates the importance weights for all assumed altitudes using measured altitude z_o, the altitude error variance σ_z^2 and the multipath altitude, which are from "Multipath propagation generator".

$\boldsymbol{w}$ means importance weight set as follows.

$$\boldsymbol{w} = \left(w_1, w_2, \cdots, w_M\right)$$

e) Resampling (Section D)

"Resampling" process resample particles with assumed altitudes based on the importance weights according to the particle filter theorem. The particles with assumed altitudes which are the top Pu percent in the importance order are resampled according to the importance weights, and random noise is added to the assumed altitudes. The rest of the particles are removed, and new particles with assumed altitudes are generated according to uniform random numbers.

f) Altitude Estimator (Section E)

"Altitude Estimator" calculates weighted average altitude using the importance weight of the assumed altitudes which are the top Pu percent in the importance order, and outputs the weighted average one as the estimated altitude of the target.

The details of the above processes from a) to f) are described as follows. In addition, Fig.4 shows the flowchart of proposed method.

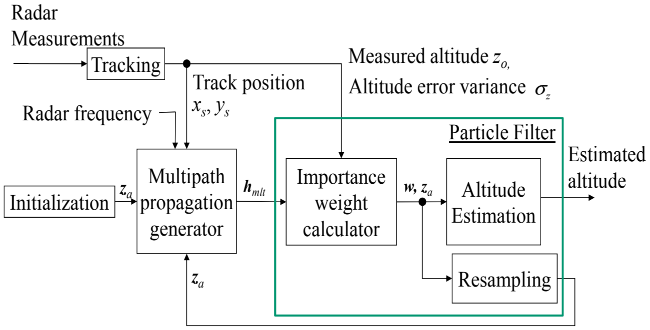

Figure 3. Block diagram of proposed method

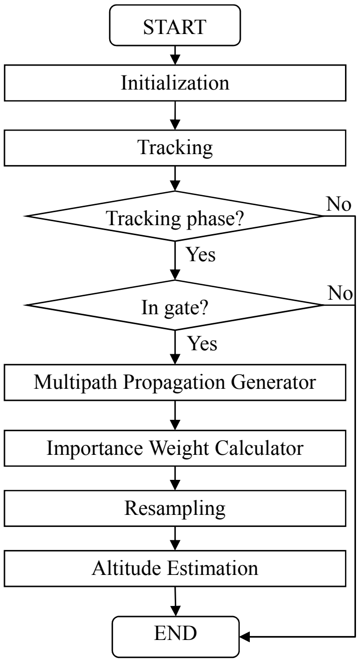

Figure 4. Flowchart of proposed method

A. Tracking

Tracking process uses two coordinate systems which are North-East-Up (NEU) coordinates and Polar (POL) coordinates (see Fig.5).

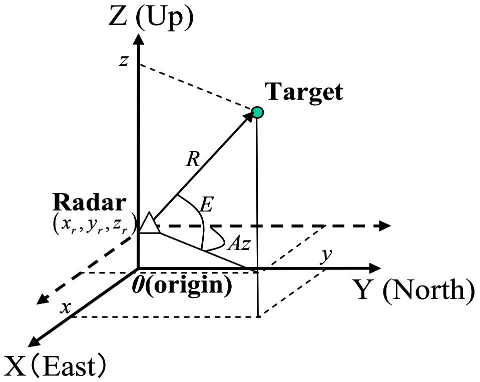

Figure 5. NEU and POL coordinates

In Fig.4, the radar position vector is defined as $\begin{bmatrix} x_r & y_r & z_r \end{bmatrix}^T$, so that the relationship between NEU and POL is defined as

$$\begin{pmatrix} R \\ E \\ Az \end{pmatrix} = \begin{pmatrix} \sqrt{(x - x_r)^2 + (y - y_r)^2 + (z - z_r)^2} \\ \tan^{-1}\left(\dfrac{z - z_r}{\sqrt{(x - x_r)^2 + (y - y_r)^2}} \right) \\ \tan^{-1}\left(\dfrac{x - x_r}{y - y_r} \right) \end{pmatrix} \quad (12)$$

Our proposed method uses tracking filter to improve the position accuracy in XY coordinates and to remove abnormal measurements which have spike errors. Tracking gate can remove the measurements which have spike errors. The tracking gate regards the measurements which satisfy (13) as "in" gate, otherwise as "out" of gate[11].

$$\boldsymbol{v}_k^T \boldsymbol{S}_k^{-1} \boldsymbol{v}_k \le d \quad (13)$$

where $\boldsymbol{v}_k$ is measurement residual (the difference between the position vector of a measurement and predicted position vector from tracking), $\boldsymbol{S}_k$ is residual covariance, and d is gate size which is determined chi-square distributed with 3 degrees of freedom.

In addition, tracking begins after at least 3 measurements are observed through all time. We call the time after tracking started "tracking phase".

B. Multipath propagation generator

The multipath altitude $h_{mlt,m}$ of an assumed altitude $z_{a,m}$ at time t_k is defined as follows.

$$h_{mlt,m(k)} = \sqrt{(x_{sk} - x_r)^2 + (y_{sk} - y_r)^2} \cdot \tan\{E_{ok}\} + z_r \quad (14)$$

$$E_{ok} = \tan^{-1}\left(\frac{z_{a,m} - z_r}{\sqrt{(x_{sk} - x_r)^2 + (y_{sk} - y_r)^2}} \right) + \Delta E_k \quad (15)$$

where the elevation angle error ΔE_k is calculated by using (1) and target position vector = $\begin{bmatrix} x_{sk} & y_{sk} & z_{a,m} \end{bmatrix}^T$. Note that, as can be seen from Fig.2, the elevation angle error ΔE_k contains spike errors and constant bias errors. "Tracking" in our proposed method has removed the spike errors before this process "Multipath propagation generator", so that the spike errors are not needed to be modeled.

C. Importance weight calculator

In case of that measured altitude z_{ok} is given at time t_k, the importance weight $w_m\ (m = 1, 2, \cdots, M)$ which corresponds to the probability of the measured altitude under the assumed altitude $z_{a,m}$[12], is given by

$$w_m = \frac{\gamma_m}{\sum_{m=1}^{M} \gamma_m} \quad (16)$$

where γ_m is defined as

$$\begin{aligned} \gamma_m &= P(z_{ok} \mid z_{a,m}) \\ &= g(z_{ok}; h_{mlt,m(k)}, \sigma_{zk}^2) = \frac{1}{\sqrt{2\pi\sigma_{zk}^2}} \exp\left(-\frac{1}{2} \frac{(z_{ok} - h_{mlt,m(k)})^2}{\sigma_{zk}^2} \right) \end{aligned} \quad (17)$$

D. Resampling

The resampling takes place with probabilities proportional to the importance weights of the top Pu percent in the importance order. Where Pu is one of the parameters in proposed method and only Pu percent of all assumed altitudes are resampled. In addition, our proposed method adds the value drawn from Gaussian random distribution with standard deviation σ_h (which is so called proposed distribution in particle filter[12]) to the resampled assumed altitude, for instance, which is j-th assumed altitude for next sampling time as follows.

$$z_{a,j(k+1)} = z_{a,j(k)} + \Delta h \quad (18)$$

where Δh is Gaussian random variable with mean 0 and variance σ_h.

Moreover, proposed method deletes the rest of assumed altitudes (which correspond to 100-Pu percent of all assumed altitudes), and generates new assumed altitudes the same number as the deleted ones. Note that new assumed altitudes are generated from uniform distribution in the interval [a,b], that is, the probability density function is given by

$$p(z) = \begin{cases} \dfrac{1}{b-a} & z \in [a,b] \\ 0 & elsewhere \end{cases} \quad (19)$$

where z is new assumed altitude for next sampling time, a is the same setting value as the lowest limit, b is the same setting value as the upper limit of the preset assumed altitude in "Initialization" process (See table Ⅱ).

E. Altitude Estimator

We calculate weighted average altitude using the importance weight of the assumed altitudes which are the top Pu percent in the importance order. The weighted average altitude is given by

$$\hat{z}=\sum_{m\in \boldsymbol{M}_t} w_m z_{a,m} \qquad (20)$$

where $\boldsymbol{M}_t$ is the index set which corresponds to the top Pu percent in the importance order.

IV. SIMULATION RESULTS

The validity of our proposed method has been examined through computer simulations as follows.

A. Simulation Scenario

Fig.6 shows the geometry in simulation scenario. In all scenarios, a target starts from the point A (40km distance from the radar) to the point B with constant horizontal speed 250m/s. The target has a constant altitude of h_t (100m, 250m and 700m) above the sea surface. The radar is located at the point C which has altitude of 4m.

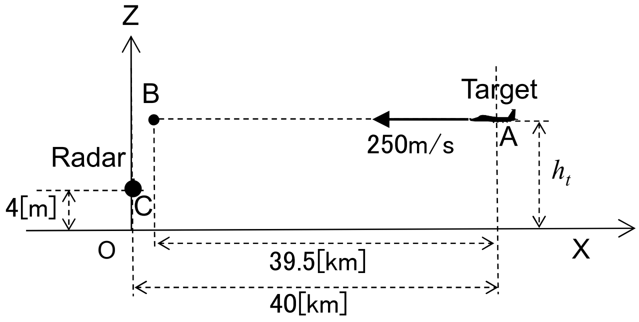

Figure 6. Simulation scenario

B. Simulation Parameters

The radar parameters are summarized in table I.

The parameters of the tracking filter are set to be as follows. The range, the azimuth and the elevation measurement noise standard deviation are 50m, 10mrad, and 10mrad respectively. The process noise standard deviation at axis x, y and z are $1\mathrm{m/s}^2$, $1\mathrm{m/s}^2$, and $0.01\mathrm{m/s}^2$ respectively. The gate size d is determined by chi-square distribution with significance level 0.01% and 3 degrees of freedom.

TABLE I. RADAR PARAMETERS

Parameter	Value
Range error standard deviation (m)	50
Azimuth error standard deviation (mrad)	10
Elevation error standard deviation (mrad)	10
Sampling rate(s)	2
False-alarm probability	10^{-6}

The parameters of the proposed method are summarized in table Ⅱ. In table Ⅱ, the assumed altitudes of a proposed method are in the range of from 100m to 600m at intervals of 0.5m and the number of those is 1001. Note that those parameters are set in “Initialization” process. Also, Pu represents the percentage of the assumed altitudes resampled from all the ones.

In addition, we compare the proposed method with the conventional method[8]. The conventional method calculates the reliability of assumed altitudes which are predetermined and fixed to output the one with highest reliability as estimated altitude. The conventional method parameters are summarized in table Ⅲ. The assumed altitudes of a conventional method are in the range of from 100m to 600m at intervals of 20m and the number of those is 26. In table Ⅲ, Th_{SNR} is threshold for deleting multipath altitudes which contain spike error as shown in Fig.2, Th_{β} and Th_{Cnt} are thresholds for selecting a sequential assumed altitude with higher reliability than specified value as estimated altitude. Th_{β} is the specified value stated above for judging the reliability, Th_{Cnt} is threshold for judging the continuity[8].

TABLE II. PROPOSED METHOD PARAMETERS

Parameter		Value
Assumed altitudes	The number M	1001
	The lowest limit (m)	100
	The upper limit (m)	600
	Interval (m)	0.5
Pu (%)		99
Standard deviation σ_h (m)		10

TABLE III. CONVENTIONAL METHOD PARAMETERS

Parameter		Value
Assumed altitudes	The number M	26
	The lowest limit (m)	100
	The upper limit (m)	600
	Interval (m)	20
False-alarm probability		10^{-6}
Th_{SNR} (dB)		3
Th_{β}		0.95
Th_{Cnt}		2

C. Results

The performance of our proposed method is evaluated by the RMSE (Root Mean Square Error) of the target position estimate in 50 Mote Carlo runs. Fig.7, 8 and 9 show the RMSE of target position estimate where the true altitude of the target is respectively 100m, 250m and 700m. In the figures, the horizontal axis represents the simulation time, the vertical axis represents the RMSE, the solid line with triangle sign represents proposed method, and the dashed line with plus sign represents conventional method.

As results, the RMSE of proposed method is about a half of that of conventional method at the time 80s in Fig.8, at the time 140s in Fig.9. However, in Fig.7, the RMSE of proposed method is almost same as that of conventional method because the true altitude of 100m is one of the preset assumed altitudes in conventional method.

From these simulation results, it is verified that the accuracy of altitude estimation is improved by the proposed method compared with the conventional method.

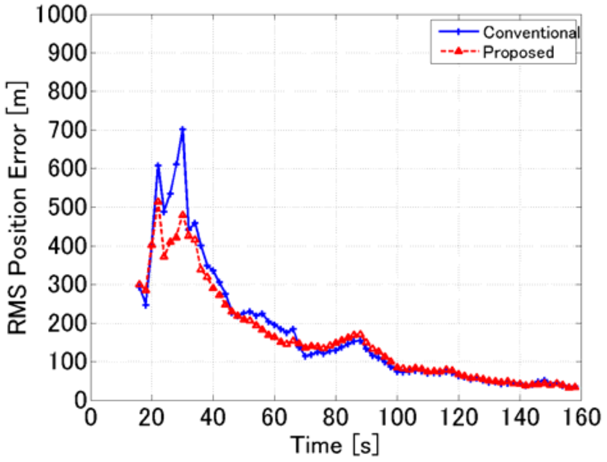

Figure 7. RMSE of position estimate (target altitude=100m).

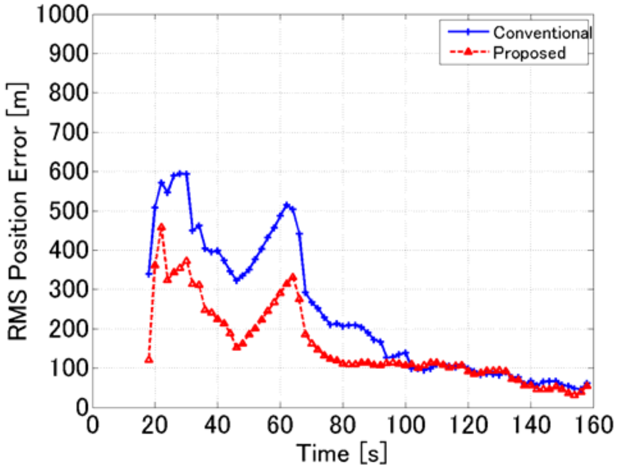

Figure 8. RMSE of position estimate (target altitude=250m).

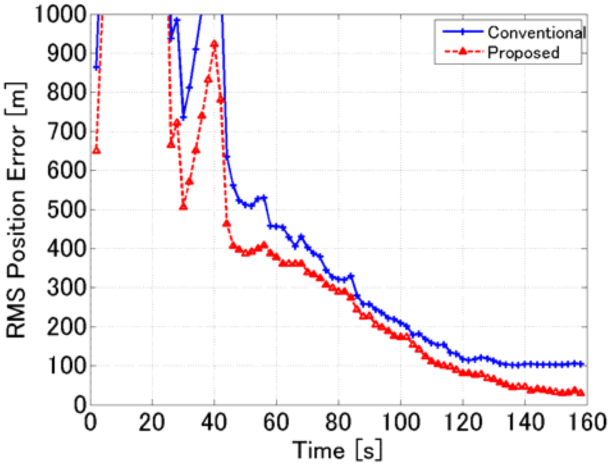

Figure 9. RMSE of position estimate (target altitude=700m).

V. CONCLUSION

In this paper, we proposed a new altitude estimation method using a particle filter for the problem of multipath propagation over the sea. A conventional method which calculates the reliability of the preset assumed altitude was not able to estimate the altitudes but the preset target altitudes. Therefore, the altitude accuracy deteriorates.

On the other hand, our proposed method calculated the importance weight of assumed altitudes, which corresponds to particles in particle filter, based on multipath propagation model to estimate target altitude. Compared with conventional method, our proposed method has the advantage of being able to estimate the altitude but the preset ones by using particle filter theorem. Simulation results showed that our proposed method reduced altitude estimation error to a half of that of conventional method, particularly in the far distance from a radar.

Through computer simulation trials, our proposed method showed the high track accuracy compared with conventional method.

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