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The Sway Compensation Trajectory for a Biped Robot

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Abstract

From 1970's, legged robots have attracted much attention of many researchers. In spite of this, it has been regarded that dynamically stable walking is very difficult to be tackled for any types of legged robots. For a trot gait for quadruped walking robots, we have proposed "the sway compensation trajectory". This method utilizes a lateral, longitudinal, and vertical motion of a robot body to keep a zero moment point (ZMP) on a diagonal line between support legs. In this paper, we develop the sway compensation trajectory for a biped robot, and show that dynamically stable walking is realized. This method makes it quite easy to design stable ZMP and COG (center of gravity) trajectories, which have been regarded as a very complicated and delicate problem. The effectiveness of the proposed method is verified through computer simulations and walking experiments by a humanoid robot, HOAP-1.

1 Introduction

A Legged robot, which has serial/parallel link mechanisms with one or more degree-of-freedoms to support own body, and repeat the intermittent contact with the ground to move, have attracted much attention of many robotics researchers from 1970's, The reason why may be that a legged robot is very appealing target to be tackled since a unique mechanism and a refined control system have to be designed and integrated.

Various types of legged robots with from one to eight legs have been developed in many research groups[1]. The advantages of these legged robots are summarized as follows:

1. Legged robots have much compatibility with existing social facilities that have been designed and constructed considering the use of the human.
2. Legged robots have latent moving performance which is higher than wheeled vehicles since they have many and redundant degree-of-freedoms.
3. The contact area with the ground is small in comparison to the sweep area of the body.
4. Contact points can be selected freely.
5. Relative position between the ground and the body is changeable while maintaining the same contact state.
6. Legs can be used as manipulators to perform various tasks.

From these features, the activity of legged robots in many field have been expected. For example, two legged humanoid robots are expected for the medical support at hospitals and welfare facilities, or daily maintenance in nuclear plants. four or six legged walking robots are also expected for the use of lumbering at forest area, constructing work at steep slope, and mine detection and removal system.

On the other hand, some drawbacks have been pointed out as compared with wheeled robots, such as,

1. Low weight capacity and low moving efficiency because of their mechanical complexity.
2. Low moving speed.
3. Body oscillation is easily induced by intermittent contact with the ground.
4. Appropriate motion control is indispensable to avoid falling down.

The quadruped is the fundamental configuration of the major of mammals. This is composed of a minimum number of legs to keep stability while statically stable walking, and to avoid complete tumbling while dynamically stable walking [2]. In addition, since the moving efficiency of walking robots depends on the number of actuators which the robot is equipped with, four legged robots have higher moving efficiency than robots with 5 or more number of legs. In conclusion, four legged robots have high superiority in practical use, and thus, various four legged robots which actualizes various types of gaits, such as crawl gait for statically stable walking, and walk, trot, pace, bound, and gallop gaits for dynamically stable walking, have been developed.

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The authors have been proposing "the sway compensation trajectory" [3],[4],[5] to realize the dynamically stable trot for four legged robots. In trot gait, two pairs of legs, that is, left front and right back legs, and right front and left back legs, repeat the contact with the ground alternatively. In case that the center of gravity (or the ZMP, more correctly) is not on a diagonal line between legs in each pair, tumbling moment along the diagonal line is produced. Thus, if this moment cannot be cancelled, the body begin to turn over and waking condition becomes unstable. The sway compensation trajectory is the method that uses a lateral, longitudinal, and vertical motion of the vehicle body to keep the ZMP on the diagonal line of the support legs.

On the other hand, the study in two legged robots has become active in recent years. Since two legged robots don't have a feature of "safe gait", which is a innate property to avoid complete tumbling[2], proper gait control is necessary to avoid turning over. Additionally, though the large support polygon connected with several support points is utilized for four legged robots, two legged robots must have a phase in which the body is supported by a single leg, and support polygon is the inside of the small sole.

Then, following two ideas have been proposed to realize stable walk for biped robots.

1. The ZMP trajectory which passes through the both soles is designed at first, then COG (center of gravity) trajectory which realizes the planned ZMP trajectory is determined by convergent calculation or spline interpolation [6],[12],[13],[14].

2. Assuming an inverted pendulum, COG trajectory with no ankle torque is obtained. Timing of the exchange of support legs and proper contact positions are determined considering the stability between adjacent COG trajectories [9],[17].

In addition, the control of torso position and acceleration, ankle torque, and contact points have been proposed[7],[15] enabling that the actual ZMP position measured by force sensors coincides with the planed ZMP trajectory. Though the category (1) is intuitive since the ZMP trajectory is a designed directly, it is often happened that the COG trajectory to realize the planned ZMP trajectory is not feasible because of the joint position/torque limitation. Thus, the ZMP trajectory must be designed carefully considering the feasible COG trajectory.

In this paper, we apply the sway compensation trajectory proposed for quadruped walking robot to a biped robot. This method can be classified as category (1): The ZMP trajectory passing through both soles, and COG trajectory which realizes the designed ZMP trajectory are both simultaneously determined by explicit functions of time. In this method, simple single mass model is assumed at first. Then the ZMP and COG trajectories which move the diagonal line between support legs is planned to connect the adjacent single leg support phase. Finally, using a multiple mass model, convergent calculation is executed so that the actual ZMP trajectory should coincide with the planned one.

This method is quite simple because the ZMP and COG trajectories which are very close to the stable state are obtained from simple explicit functions of time. In addition, same explicit functions is applicable for various two legged robots which have different shapes and configurations of degree-of-freedoms. Thus, this method can is highly flexible. Moreover, since approximately stable trajectories considering a multiple mass model are obtained, practical dynamically stable gait will be realized by combining the proposed method with the feedback control using torso position/acceleration and ankle torques.

This paper is organized as follows: the procedure to obtain the ZMP and COG trajectories by the sway compensation trajectory is explained in section 2. In section 3, several results of computer simulations and experiments using a humanoid robot are introduced.

2 The sway compensation trajectory for biped robots

In this section, a new sway compensation trajectory for biped robots is introduced. Unlike the trajectories proposed for quadruped robots [3],[4],[5]. The ZMP and COG trajectories are obtained by explicit functions of a sway width, a walking speed, acceleration and lines between the support legs.

2.1 Derivation of the sway compensation trajectory

In the following discussion, we assume the weight of legs and arms are smaller than the body weight. Then, the system can be regarded as a single mass model placed on (x_g, y_g, z_g)

Assuming a floor is flat and the height of the point mass z_g is constant, the ZMP on the floor $(x_{zmp}, y_{zmp}, 0)$ is obtained as follows:

$$\begin{pmatrix} x_{zmp} \\ y_{zmp} \end{pmatrix} = \begin{pmatrix} x_g \\ y_g \end{pmatrix} - A \begin{pmatrix} \ddot{x_g} \\ \ddot{y_g} \end{pmatrix} \quad (1)$$

where, $A = \frac{z_g}{g}$. Next, the diagonal line of the support legs is defined as

$$x\ \cos\theta + y\ \sin\theta = d \quad (2)$$

Then, in order to keep the ZMP on this line, the center of gravity has to satisfy the next equations.

$$\cos\theta(x_g - A\ddot{x_g}) + \sin\theta(y_g - A\ddot{y_g}) = d \quad (3)$$

Here, we assume that the robot moves along x axis and the position of the body is expressed as

$$x_g = C_1^x e^{\frac{t}{\sqrt{A}}} + C_2^x e^{-\frac{t}{\sqrt{A}}} + a_2^x t^2 + a_1^x t + a_0^x \quad (4)$$

$$y_g = C_1^y e^{\frac{t}{\sqrt{A}}} + C_2^y e^{-\frac{t}{\sqrt{A}}} + a_2^y t^2 + a_1^y t + a_0^y \quad (5)$$

Eqs.(4) and (5) consists of a particular solution and a general solution of equations, $x_g - A\ddot{x_g} = 0$ and $y_g - A\ddot{y_g} = 0$. By substituting these equations into Eq.(3), we get

$$\cos\theta(a_0^x - 2Aa_2^x + a_1^x t + a_2^x t^2)$$

$$+\sin\theta(a_0^y - 2Aa_2^y + a_1^y t + a_2^y t^2) = d \quad (6)$$

To satisfy the above equation at any time, we get

$$\cos\theta(a_0^x - 2Aa_2^x) + \sin\theta(a_0^y - 2Aa_2^y) = d \quad (7)$$

$$\cos\theta a_1^x + \sin\theta a_1^y = 0 \quad (8)$$

$$\cos\theta a_2^x + \sin\theta a_2^y = 0 \quad (9)$$

thus,

$$a_0^x = \frac{d - \sin\theta a_0^y}{\cos\theta} \quad (10)$$

$$a_1^x = -\tan\theta a_1^y \quad (11)$$

$$a_2^x = -\tan\theta a_2^y \quad (12)$$

Next, other parameters are determined from the boundary condition to keep the continuity of the trajectory. From the continuity along y direction, $\dot{y}_{g,t=0} = \dot{y}_{g,t=\frac{T}{2}} = 0$, and the ZMP sway width is D, each coefficient is determined as

$$C_1^y = \frac{8D(e^{\frac{T}{2\sqrt{A}}} - 1) - a_2^y(e^{\frac{T}{2\sqrt{A}}} + 1)T^2}{2(e^{\frac{T}{\sqrt{A}}} - 1)T}\sqrt{A} \quad (13)$$

$$C_1^y = \frac{8D(e^{\frac{T}{2\sqrt{A}}} - 1) + a_2^y(e^{\frac{T}{2\sqrt{A}}} + 1)T^2}{2(e^{\frac{T}{\sqrt{A}}} - 1)T}\sqrt{A}e^{\frac{T}{2\sqrt{A}}} \quad (14)$$

$$a_0^y = 2Aa_2^y + D \quad (15)$$

$$a_1^y = -\frac{4D}{T} - \frac{a_2^y T}{2} \quad (16)$$

In addition, in case that the robot is accelerated along x axis, and the velocity and acceleration at $t = 0$ and $\frac{T}{2}$ are $(v, a)_{t=0}$ and $(v + a\frac{T}{2}, a)_{t=\frac{T}{2}}$, we get the following equations using the boundary condition of $x_{g,t=\frac{T}{2}} - x_{g,t=0} = \frac{L}{2}$, $\dot{x}_{g,t=0} = v$, $\dot{x}_{g,t=\frac{T}{2}} v + a\frac{T}{2}$, and $\ddot{x}_{g,t=\frac{T}{2}} = \ddot{x}_{g,t=0} = a$.

$$C_1^x = \frac{a + 2a_2^y\tan\theta}{4(e^{\frac{T}{2\sqrt{A}}} - 1)}\sqrt{A}T \quad (17)$$

$$C_2^x = \frac{a + 2a_2^y\tan\theta}{4(e^{\frac{T}{2\sqrt{A}}} - 1)}\sqrt{A}Te^{\frac{T}{2\sqrt{A}}} \quad (18)$$

$$T = \frac{2(-v + \sqrt{v^2 + 4aD\tan\theta})}{a} \quad (a \neq 0) \quad (19)$$

$$= \frac{4D\tan\theta}{v} \quad (a = 0) \quad (20)$$

$$(21)$$

$$L = 4D\tan\theta \quad (22)$$

In Eqs.(10)∼(22), manipulatable parameters are v, a, and a_2^y. The trajectory expressed by Eqs.(4) and (5) is named "the sway compensation trajectory". The ZMP trajectory is also derived by substituting Eqs.(4) and (5) into Eq.(1).

Next, we derive the condition to maintain the ZMP on the sole of the support leg in single support phase. Here, we consider the ZMP in lateral (y) direction only, since the case that the ZMP goes out from the sole toward longitudial (x) direction is considered to be very rare if the stride is practically small.

As shown in Fig.1, the inside edge of the sole is at αD. The time to reach the y component of the ZMP to this edge is obtained from $y_{zmp}(t) = \alpha D$ as

$$t = \frac{1 - \alpha}{4}T \quad (23)$$

Therefore, as shown in Fig.2, the condition to maintain the

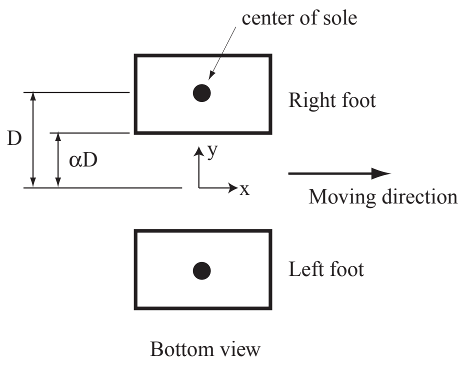


Figure 1: Condition to keep the ZMP in soles

ZMP on the sole of the support leg in single support phase is derived from

$$\frac{1}{2} - \frac{2\beta - 1}{4} < \frac{1 - \alpha}{4} + \frac{1}{4} \quad (24)$$

as

$$1 > \beta > \frac{\alpha + 1}{2} \tag{25}$$

where, β is a duty factor.

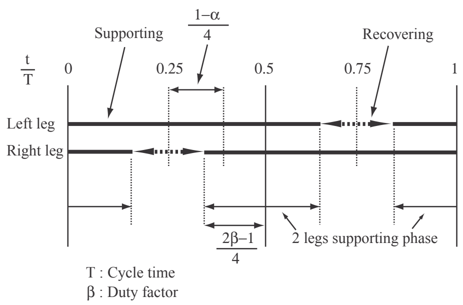


Figure 2: Chart of leg phases

Fig.3 shows an example of the ZMP and COG trajectories in case that the robot walks 10 steps with the velocity $0.1 m/s$ align x axis from inital position $(0, 0)$.

It should be pointed out that obtained trajectories can be utilized if the moving direction is gradually changed. That is to say, continuous ZMP and COG trajectories are obtained by applying original trajectories with simple coordinate transformation according to the moving direction. Fig.4 shows the case that the moving direction is changed along with the curved path.

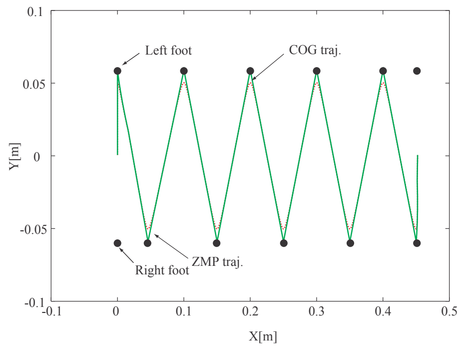


Figure 3: An example of ZMP and COG trajectories for straight path

2.2 Convergent calculation considering multiple mass model

The trajectories obtained as Eqs.(4) and (5) are based on the assumption of a simple single mass model. Therefore, more precise trajectories and joint commands should be determined based on a multiple mass model to realize a more stable walk. In this section, we propose a new method that

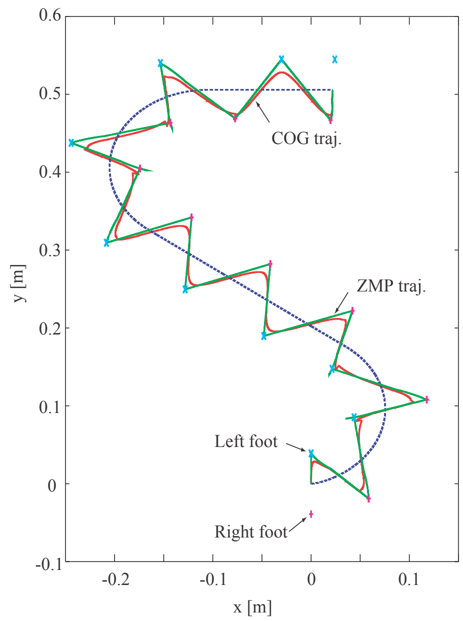


Figure 4: An example of ZMP and COG trajectories for curved path

enable the precise ZMP trajectory derived from a multiple mass model to coincide with the planned ZMP trajectory based on a single mass model. This method utilizes the convergence calculation of rectification of COG trajectory according to the current ZMP errors.

Firstly, the ZMP $X_{zmp} = (x_{zmp}, y_{zmp}, 0)$ for a multiple mass model is obtained from the next equations.

$$x_{zmp} = \frac{\sum_{i=1}^{N} m_i(r_{ix}(\ddot{r}_{iz} + g) - r_{iz}\ddot{r}_{ix}) - I_i\dot{\omega}_{iy}}{\sum_{i=1}^{N} m_i(\ddot{r}_{iz} + g)} \tag{26}$$

$$y_{zmp} = -\frac{\sum_{i=1}^{N} m_i(-r_{iy}(\ddot{r}_{iz} + g) + r_{iz}\ddot{r}_{iy}) - I_i\dot{\omega}_{ix}}{\sum_{i=1}^{N} m_i(\ddot{r}_{iz} + g)} \tag{27}$$

Here, we define the planned ZMP trajectory as X_{zmp}^{ref}, the refined COG trajectory to realize the ZMP trajectory as X_g^{ref}, and the current COG trajectory as X_g. Then, current situation can be expressed as

$$X_{zmp} = X_g - A\ddot{X}_g \tag{28}$$

On the other hand, the goal situation is shown as

$$X_{zmp}^{ref} = X_g^{ref} - A\ddot{X}_g^{ref} \tag{29}$$

Thus, substructing above equations, we get

$$X_{zmp}^{ref} - X_{zmp} = X_g^{ref} - X_g - A(\ddot{X}_g^{ref} - \ddot{X}_g) \tag{30}$$

thus,

$$e_{zmp} = e_g - A\ddot{e}_g \tag{31}$$

where, $e_{zmp} = X^{ref}_{zmp} - X_{zmp}$, and $e_g = X^{ref}_g - X_g$. By resampling this equation with sampling interval Δt, next equation is obtained.

$$\begin{aligned} e^t_{zmp} &= e^t_g - A\ddot{e}^t_g \\ &= e^t_g - A\frac{e^{t+1}_g - 2e^t_g + e^{t-1}_g}{(\Delta t)^2} \end{aligned} \tag{32}$$

Thus, the rectified value of COG trajectory e^t_g that make the ZMP error e^t_{zmp} to be 0 is obtained as the next equation.

$$e^t_g = \frac{e^t_{zmp} + \frac{A}{(\delta t)^2}(e^{t+1}_g + e^{t-1}_g)}{1 + 2\frac{A}{(\delta t)^2}} \tag{33}$$

However, several experiments showed that the refined COG trajectory tend to be oscillational at discontinuous point of the ZMP velocity, if the rectified value obtained above equation is added directly to the original COG trajectory. Therefore, we choose the rectified value of COG trajectory e^t_g so that the next equation appending the smoothness constraint with Eq.(33) is minimized.

$$\min_{e^t_g} \mid e^t_g - \frac{e^t_{zmp} + \frac{A}{(\delta t)^2}(e^{t+1}_g + e^{t-1}_g)}{1 + 2\frac{A}{(\delta t)^2}} \mid + k \mid e^t_g - \frac{e^{t+1}_g + e^{t-1}_g}{2} \mid \tag{34}$$

Consequently, the refined COG trajectory that realizes the planned ZMP trajectory is obtained as the sum of current COG trajectory X_g and the rectified value e^t_g as

$$X^{ref}_g \leftarrow X_g + e^t_g \tag{35}$$

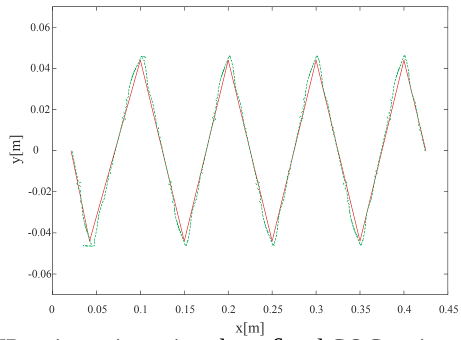
In practice, considering the resampling error and non linearity of Eqs.(28),(29), the above calculation should be repeated until e^t_g becomes sufficiently small value.

Fig.5 shows the planned and calculated ZMP trajectories for a humanoid robot HOAP-1 which walks along x axis. Fig.5(a) shows the ZMP trajectories with convergence calculation, and (b) is the original trajectory without convergence calculation. And Fig.6 shows the COG trajectories before and after convergence calculation,

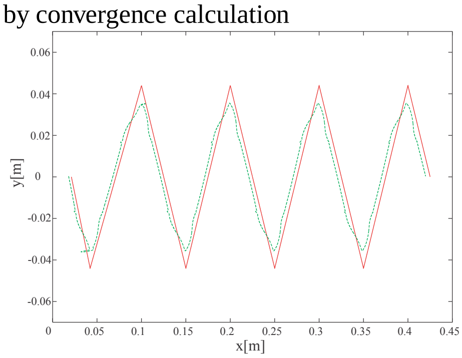
3 Experiment

3.1 Computer simulation

Computer simulation is carried out on the dynamic simulator, OpenHRP [18],[19],[20], which has being developed at National Institute of Advanced Industrial Science and Technology (AIST) and The University of Tokyo.



(a) ZMP trajectories using the refined COG trajectory obtained by convergence calculation



(b) ZMP trajectories using the original COG trajectory

Figure 5: Refined ZMP trajectories

An example of simulation results is shown in Fig.7. In this example, the robot increases the velocity from a stationary state at first. Then, after steadily walking state with a stride of 0.2m, the robot reduces the velocity and stops again. Total simulation time is 40 seconds. In this simulation, command angles for each joint is calculated before executing simulation, and thus, any on-line feedback controls such as the use of ankle torque aren't applied.

From several computer simulations, it was proven that the stable walk of the biped robot is possible by the proposed ZMP and COG trajectories.

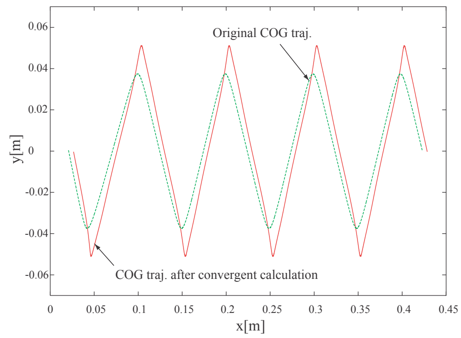


Figure 6: Refined COG trajectories

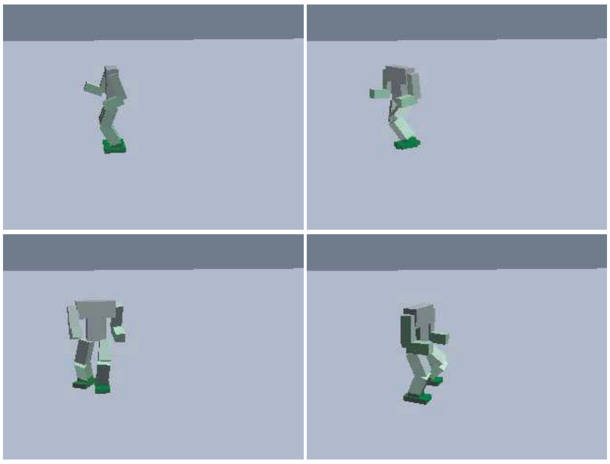


Figure 7: An example of simulation results using OpenHRP

3.2 Experiment using a humanoid robot

Walking experiments using a humanoid robot HOAP-1 (Fujitsu Co.Ltd) are carried out. The aspect of the walking experiment and the measured ZMP trajectory are shown in Figs.8, 9, and 10. In this experiment, command angles for each joint is predetermined, and on-line feedback control isn't applied.

As shown in Fig.8, the robot fell down just after starting walking motion without the proposed convergent calculation. However, by using the trajectories given by the proposed convergent calculation, it kept walking without losing the balance as shown in Fig.9. From these experiments, it was verified that the stable biped walk is successfully performed by the proposed ZMP and COG trajectories.

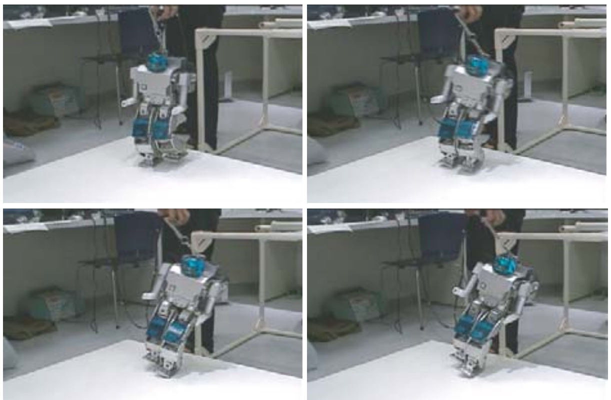


Figure 8: An example of walking experiments using HOAP-1 before convergent calculation (Walking speed 0.1m/s, Duty factor 0.75). The robot fell down just after starting walking motion.

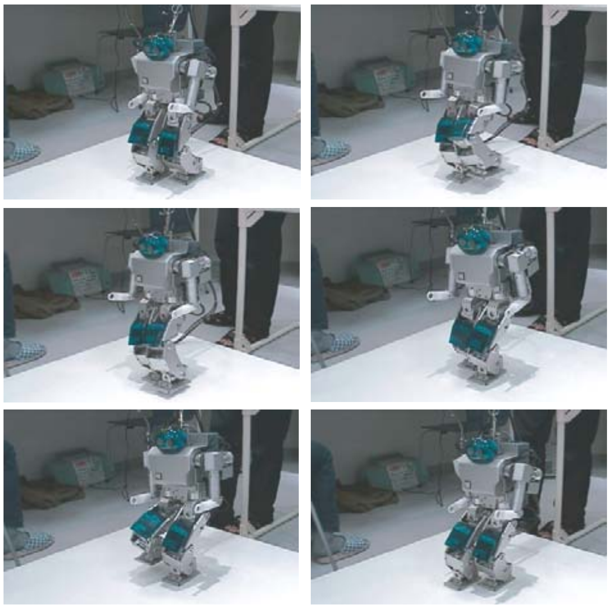


Figure 9: An example of walking experiments using HOAP-1 after convergent calculation (Walking speed 0.1m/s, Duty factor 0.75)

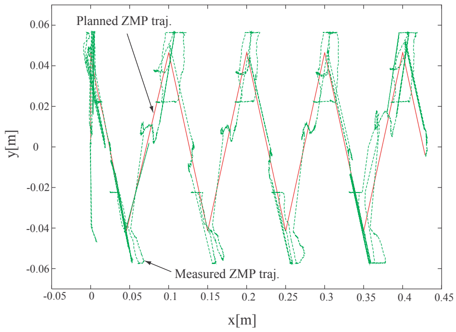


Figure 10: Measured ZMP trajectories

4 Conclusions

In this paper, a sway compensation trajectory for biped robots is proposed. The ZMP trajectory passing through both soles, and COG trajectory which realizes the designed ZMP trajectory are both simultaneously determined by explicit functions of time.

In this method, simple single mass model is assumed at first. Then ZMP which moves the diagonal line between support legs is planned to connect the adjacent single leg support phase. Finally, using multiple mass model, convergent calculation is executed so that the actual ZMP trajectory coincides woth the planned one. This method is quite simple because the ZMP and COG trajectories which are very close to the stable state are obtained by simple explicit functions of time. In addition, same explicit functions is applicable for various two legged robots which have different shapes and configurations of degree-of-freedoms, and thus, this method can be regarded as being highly flexible.

We are now developing the practical biped robot control system which combines the proposed method with the feedback control using torso position/acceleration and ankle torques.

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