An Experimental Study of a Cooperative Positioning System

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Abstract. Several position identification methods are being used for mobile robots. Dead reckoning is a popular method but due to the error accumulation from wheel slippage, its reliability is low for measurement of long distances especially on uneven surfaces. Another popular method is the landmark method, which estimates current position relative to known landmarks, but the landmark method's limitation is that it cannot be used in an uncharted environment. Thus, this paper proposes a new method called "Cooperative Positioning System (CPS)" that is able to overcome these shortcomings. The main concept of CPS is to divide the robots into two groups, A and B where group A remains stationary and acts as a landmark while group B moves and then group B stops and acts as a landmark for group A. This process is repeated until the target position is reached. Compared with dead reckoning, CPS has a far lower accumulation of positioning errors, and can also work in three dimensions. Furthermore, CPS employs inherent landmarks and therefore can be used in uncharted environments unlike the landmark method. In this paper, we introduce the basic concept of CPS and its positioning principle. Next, we outline a second prototype CPS machine model (CPS-II) and discuss the method of position estimation using the variance of positioning error and weighted least squares method. Position identification experiments using the CPS-II model give a positioning accuracy of 0.12% for position and 0.32 degree for attitude after the robots traveled a distance of 21.5 m. .

Keywords: position identification, multiple robots, cooperation, outdoor environment

1. Introduction

Control of mobile robots traveling in an uncharted environment necessarily requires a method of identifying the position of the robots. A number of position identification techniques for mobile robots have so far been proposed. The methods can be roughly classified into two types: dead reckoning and landmark.

The dead reckoning method identifies robot position by calculating the amount of travel from the starting point. One type of the method executes the task by integrating rotations of the right and left wheels, in the case of wheel-driven vehicles. This odometry is simple and is easily implemented. It can also be used to identify positions in uncharted environments as it only employs internal sensors.

The dead reckoning method, however, has a serious problem. Wheel slippage or sensor drift may cause measurement errors which accumulate while the vehicle travels. The odometry that only measures the rotation of wheels does not work in threedimensional terrains including cases involving difference of elevation. Several modified methods of odometry to measure 3D undulations have been proposed until now [1],[2], [3], but it is still very difficult to measure 3D locomotion precisely.

The landmark method [4], [5], [6], [7], [8], [9], [10], [11], [12], [13] identifies the position of a mobile robot by measuring the current position relative to landmarks at identified positions. There are many types of landmark methods depending on the type of landmark and how it is placed. The use of the shape of the horizon line [14], [15] and stationary satellites (GPS) [16],[17],[18] are classified under the landmark method. This method features high measurement accuracy with no accumulation of measurement error. It also provides three-dimensional information including elevation. However, it has a great drawback in that the environment around the mobile robot must be known and specified landmarks must be placed in the environment in advance.

In this paper, we propose a new method called "Cooperative Positioning System with multiple robots (CPS)" [22],[23],[24]. Instead of using landmarks in the environment, this technique uses the robots themselves as mobile landmarks where they use each other as landmarks and exchange position information. This gives accurate positioning even in uncharted environments. The basic idea underlying this technique is as follows: Each of the robots repeats move-and-stop actions and serves as a landmark for the other robots. In this way, the entire group of the multiple robots travels while maintaining knowledge of their respective positions.

There has been a great deal of interest in using autonomous multiple mobile robots to improve efficiency and robustness in performing a task. Also, the measurement of relative position and mutual situation between mobile robots has been studied in some groups [25],[26],[27]. For example, Arai et al. [25] proposed the system which measures the mutual relative position between mobile robots using CCD camera and LED. Yuta et al. [26] developed a wireless communication system to inform the current position of each other. However, these works treated mainly cooperative task planning or collision detection, and high-precision position identification as a group of robots has not been comprehensively considered. Our study focuses on the nature of multiple robots as a group and uses it for high precision position identification. This idea has not been studied in depth before.

In this paper, we first discuss the basic principles of the proposed CPS. Next, we outline a new second prototype CPS machine model (CPS-II) and discuss a method of position estimation that enable us to perform appropriate integration of redundant position information using the variance of positioning error and weighted least squares method. We then report the results of position identification experiments using the model.

2. Cooperative Positioning System (CPS)

2.1. Standard CPS Configuration

The proposed cooperative positioning system (CPS) with multiple robots may be referred to as a mobile landmark method. In this section, we define the basic principles of CPS clearly and show examples of standard CPS configuration. Robots were divided into two groups of A and B.

- 1. While robot group A with its initial position identified remains stationary, robot group B travels a certain distance. The position of traveling group B is roughly determined by measuring its position relative to group A and using internal sensors as well.
- 2. After group B travels an arbitrary distance, its position is accurately measured in reference to the position of group A.
- 3. Group B then remains stationary while group A travels an arbitrary distance.
- 4. Steps 1 to 3 are repeated until the target position of the robot group is reached.

Figs. 1 and 2 show how the positioning is actually performed with CPS. Fig. 1 shows an example using two robots. When position $P_1(x_1, y_1, z_1)$ of the stationary robot 1 is known, CPS is implemented as follows: First, the three-dimensional position $P_2(x_2, y_2, z_2)$ of robot 2 that moved and stopped is determined, as indicated by the solid line. This is done by measuring the distance rbetween the robots, azimuth angle ϕ of a turn against the vertical axis along the direction of gravity, and elevation angle ψ from the level surface and assigning these measurements in the following equations:

$$x_2 = x_1 + r\cos\psi\cos\phi \tag{1}$$

$$y_2 = y_1 + r\cos\psi\sin\phi \tag{2}$$

$$z_2 = z_1 + r \sin \psi \tag{3}$$

Next, robot 1 moves as indicated by the dotted line. The position where robot 1 stops is determined in reference to the position of robot 2. These operations are repeated.

Fig. 2 shows an example using three robots. When positions $P_1(x_1, y_1, z_1)$ of robot 1 and $P_2(x_2, y_2, z_2)$ of robot 2 are known, CPS is implemented as follows: First, position $P_3(x_3, y_3, z_3)$ of robot 3 that moved and stopped is determined, as indicated by the solid line. This is done by measuring azimuth angles ϕ_1 and ϕ_2 and elevation angles ψ_1 and ψ_2 of robot 3 as seen from robots 1 and 2 and assigning these measurements in the following equations:

$$x_3 = \frac{x_1t_1 - x_2t_2 + (y_2 - y_1)t_1t_2}{t_1 - t_2} \qquad (4)$$

$$y_3 = \frac{-(x_2 - x_1)t_1t_2 + y_1t_1 - y_2t_2}{t_1 - t_2} \qquad (5)$$

$$z_{3} = z_{1} + \frac{t_{2}}{c_{1}(t_{1} - t_{2})} r \tan \psi_{1}$$

$$= z_{2} + \frac{t_{1}}{c_{2}(t_{1} - t_{2})} r \tan \psi_{2}$$
(6)

where

$$t_n = \tan \phi_n, \quad c_n = \cos \phi_n \quad (n = 1, 2)$$
(7)

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
 (8)

Next, robot 1, for instance, moves as indicated by the dotted-line arrow. The position where robot 1 stops is determined in reference to the positions of robot 2 and 3. These operations are repeated.

CPS is not limited to these examples; many other configurations are possible. For instance, the position of one robot relative to two stationary robots in Fig. 2 may be determined differently. That is, instead of being identified by a combination of azimuth angle ϕ and elevation angle ψ , the determination may be based on a combination of



Fig. 1. Type 1 positioning principle



Fig. 2. Type 2 positioning principle

distance r between robots and elevation angle ψ . Another CPS, unlike the one that classifies robots into two groups, may cyclically select stationary robots from a group of robots to allow the entire group to move continuously. Another CPS may determine more accurate positions based on redundant measurements from a number of stationary robots.

2.2. CPS Characteristics

CPS position identification has the following characteristics:

- 1. CPS determines the position by repeated measurements and therefore accumulates positioning errors as with the dead reckoning method. However, CPS has far fewer positioning errors because it can accurately determine position and attitude by measuring stationary points in the same way as in surveying. CPS provides a good basis for extraordinarily higher positioning accuracy than does the dead reckoning method based on wheel rotation.
- 2. Unlike the landmark method, CPS does not require prior placing of landmarks. It allows movement in uncharted environments or even under the ground where Global Positioning System (GPS) cannot be used.
- 3. By measuring elevation angles, CPS can determine three-dimensional positions, which is not possible with dead reckoning.

In contrast, CPS has the following problems:

- 1. At least two robots are required.
- 2. At any time, at least one robot must be stationary, which slows down the overall speed.
- 3. An accurate measuring device (including equipment to measure the attitude relative to gravity) must be built into each robot.
- 4. The robot that identifies its own position must be in view of several other robots.

Items 1 and 2 are not problems if an application essentially uses multiple robots and does not require high-speed movement. Item 3 depends on technological innovation in the field of measuring instruments, which has been rapidly progressing. The authors believe this problem will soon be solved. The performance of the prototype measuring instrument we assembled will be discussed later. Item 4 will become a serious problem especially when robots move in outdoor environments where many unknown obstacles exist and where they need to move very long distances. But in case that a number of robots are used, CPS positioning is possible if only a small portion of the robots in the group can be seen from the reference robot and thus we hope this limitation will not become a fatal problem.

2.3. Fields of CPS Applications

We believe that CPS that has the foregoing characteristics can be used effectively if proper applications are selected. Two possible applications are discussed below.

One is the automatic control system of a robot that cleans the inside of train stations or underground markets as shown in Fig. 3. This robot could clean a wide area based on a map. Dead reckoning based on wheel rotation has had some problems for this application. One of these problems is errors in movement measurement caused by slippery floors. Another problem is the substantial azimuth measurement errors likely to occur when the cleaning robot runs into a wall or pillar. In addition, a cleaning robot would probably need to go up and down stairs and clean floors that have uneven surfaces. As stated in item 3 above, dead reckoning, which cannot work for three-dimensional measurement, is inappropriate for this application. The landmark method is not effective, either, because landmarks will not necessarily be placed in advance in many environments. Meanwhile GPS which is remarkably effective outdoors, cannot be used for cleaning indoors.

CPS is an effective method for cleaning indoors that have uneven surfaces and many obstacles. Furthermore, in this application, it is effective to use multiple cleaning robots and all that is required is to equip each robot with a measuring device.

The point to be specifically noted for CPS is that any disturbance of the cleaning robots while working, such as an collision with a obstacle, has no effect on positioning accuracy.

A system configuration consisting of cleaning robots and a dedicated measurement robot is also a possibility. Each cleaning robot is equipped with only a marker light source. The dedicated measurement robot is equipped with sensors that measure the distance to and the angle of the marker light source of each cleaning robot. This configuration provides an inexpensive implementation of CPS.

Another possible application is the use of robots to explore unknown environments, such as planets as shown in Fig. 4. For this type of investigation, use of many small robots is most effective to take advantage of the high reliability of a redundant system. Some researchers have previously advocated the effectiveness of this multiple robots approach [28]. The authors also have proposed a new concept of multiple planetary rovers "Gunryu" and developed a prototype [29] as shown in Fig. 5. The "Gunryu" consists of multiple mobile robots with manipulators. These robots usually operate individually. But when they move over an uneven surface, they are connected to one another using the manipulators. Suppose the multiple robots such as the "Gunryu" move around over an uncharted planet to perform a geological survey or make a detailed topographical map. For position identification, both the dead reckoning and landmark methods are inappropriate from the standpoint of required precision and environmental conditions. In contrast, CPS has almost no problems with this application and is essentially effective. Furthermore, each robot of the "Gunryu" must come close to one another for connection or break away from one another for separation. Therefore, it is assumed that each robot is equipped with a relative position measuring sensor. CPS can thus be used for "Gunryu" as is without adding any new measuring devices.



Fig. 3. CPS application for cleaning robots



Fig. 4. CPS application for planetary rovers



 $Fig.\ 5.$ Cooperative multiple robots "Gunryu" in planet exploration

3. Second Prototype CPS Machine Model, CPS-II

This section outlines a second prototype CPS machine model (CPS-II). We then propose the method of position estimation using the variance of positioning error and the weighted least squares method that enable the appropriate fusion of redundant position information according to the positioning accuracy of each robot.

CPS-II consists of a parent robot equipped with a high-precision laser range finder and child robots each equipped with corner cubes. This prototype system is capable of :

- Searching for other robots.
- Measuring the distance to and azimuth angle of other robots.
- Automatically moving each robot based on the measurement results.

3.1. Outline of CPS-II

Fig. 6 shows our experimental system. CPS-II consists of one parent robot, 0, and two to three child robots, 1, 2, and 3. The parent robot (Fig. 7) is equipped with a laser range finder of TOP-CON Ltd. (Table 1) that is capable of searching and tracing a corner cube in an arbitrary position automatically and a 2-axis inclinometer. By detecting the laser reflected from the child robots,



Fig. 6. Total view of the mechanical model CPS-II.

the parent robot automatically and accurately measures the distances from the child robots and the azimuth angles. On top of each child robot are six corner cubes arranged at intervals of 60 degrees around the vertical axis. This mechanism can accurately reflect a laser beam projected from any direction. Each robot has a built-in microcomputer (8086-8MHz, Japan System Design Co., Ltd.), driving circuit, battery (Yuni-Z, YUASA BATTERY Co., Ltd.), and communication system (HRF-600 (RS-232C), HERUTU Co., Ltd.), and is controlled centrally from the host computer (S-4/Liea, Fujitsu Ltd.).

3.2. Principles of Measurement in CPS-II

In case one parent robot, 0, and two child robots, 1 and 2 are used, they move while alternately performing position identification according to the following procedure: First, the initial position and attitude of robot 0 are measured, then

- 1. Robots 1 and 2 move and then stop.
- 2. Robot 0 measures the distance r_1 , azimuth angle ϕ_1 , and elevation angle ψ_1 relative to



Fig. 7. The master robot of CPS-II.

Table 1.	Range	finder	specifications
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AP-L1	(TOPCON	Ltd.)
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Range	$4 \sim 400 [m]$
Distance resolution	$0.2 \ [mm]$
Angle resolution	5 ["]
Distance precision	±3+2ppm [mm]
Angle precision	±5 ["]



Fig. 8. Experimental CPS system

robot 1, and calculates the position of robot 1 using Equations (1), (2) and (3).

- 3. Similarly, robot 0 measures the distance r_2 , azimuth angle θ_2 , and elevation angle ϕ_2 relative to robot 2 and calculates the position of robot 2.
- 4. Robot 0 moves and then stands still.
- To calculate the position and attitude of robot 0, robot 0 measures the distances r₁ and r₂, azimuth angles φ₁ and φ₂, and elevation angles ψ₁ and ψ₂ relative to robots 1 and 2.
- 6. Return to step 1.

The above operation cycle is repeated until the entire robot group reaches the target position.

In step 5, the position of robot 0 is calculated with redundancy because the number of applicable equations, 6, is more than the number of unknown parameters $(x_0, y_0, z_0, \theta_0)$, and also if more than two child robots are used for parent robot positioning. We show the weighted least squares method using the variance of positioning error that enable systematic fusion of redundant position information and the calculation of accumulated positioning error of each robot.

Equations that are established (observation equations) in Step 5 are

$$(x_0 - x_i)^2 + (y_0 - y_i)^2 = r_i^2 \cos^2 \psi_i \qquad (9)$$

$$z_0 = z_i - r_i \sin \psi_i \tag{10}$$

$$\theta_0 = -\phi_i + \tan^{-1} \frac{y_i - y_0}{x_i - x_0} \tag{11}$$

for i = 1 and 2. If the absolute position of robot i is $\tilde{\mathbf{P}}_i(\tilde{x}_i, \tilde{y}_i, \tilde{z}_i, \tilde{\theta}_i)$ but is measured as $\mathbf{P}_i = (\tilde{x}_i + dx_i, \tilde{y}_i + dy_i, \tilde{z}_i + dz_i, \tilde{\theta}_i + d\theta_i)$ then from the Taylor expansion of above equations

$$a_i dx + b_i dy = (r_i \cos \psi_i - d_i)$$

$$+a_i dx_i + b_i dy_i + \cos \psi_i dr_i - r_i \sin \psi_i d\psi_i \quad (12)$$

 $dz = \tilde{z}_i - r_i \sin \psi_i - \tilde{z}_0 + dz_i - \sin \psi_i dr - r_i \cos \psi_i d\psi_i$ (13)

$$-\frac{b_i}{d_i}dx + \frac{a_i}{d_i}dy - d\theta_0 = \left(\tilde{\phi}_i + \tilde{\theta}_0 - \tan^{-1}\frac{\tilde{y}_i - \tilde{y}_0}{\tilde{x}_i - \tilde{x}_0}\right)$$
$$-\frac{b_i}{d_i}dx_i + \frac{a_i}{d_i}dy_i + d\phi_i \tag{14}$$

can be obtained by assuming the errors are small and the second and higher order terms may be disregarded, where $d_i = \sqrt{(\tilde{x_i} - \tilde{x_0})^2 + (\tilde{y_i} - \tilde{y_0})^2}$, $a_i = -\frac{\tilde{x_i} - \tilde{x_0}}{d_i}$ and $b_i = -\frac{\tilde{y_i} - \tilde{y_0}}{d_i}$. Next, we define

$$\mathbf{A} = \begin{pmatrix} a_1 & b_1 & 0 & 0 \\ a_2 & b_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{b_1}{d_1} & \frac{a_1}{d_1} & 0 & -1 \\ -\frac{b_2}{d_2} & \frac{a_2}{d_2} & 0 & -1 \end{pmatrix}$$
(15)
$$\begin{pmatrix} (r_1 \cos \phi_1 - d_1) \\ (r_2 \cos \phi_2 - d_2) \\ \vdots & \vdots & \ddots \end{pmatrix}$$

$$\mathbf{L} = \begin{pmatrix} \tilde{z}_{1} - r_{1} \sin \psi_{1} - \tilde{z}_{0} \\ \tilde{z}_{1} - r_{2} \sin \psi_{2} - \tilde{z}_{0} \\ \tilde{z}_{2} - r_{2} \sin \psi_{2} - \tilde{z}_{0} \\ \phi_{1} + \tilde{\theta}_{0} - \tan^{-1} \frac{\tilde{y}_{1} - \tilde{y}_{0}}{\tilde{x}_{1} - \tilde{x}_{0}} \\ \phi_{2} + \tilde{\theta}_{0} - \tan^{-1} \frac{\tilde{y}_{2} - \tilde{y}_{0}}{\tilde{x}_{2} - \tilde{x}_{0}} \end{pmatrix}$$
(16)

and substitute dx_i , dy_i and dz_i as 0 by assuming previous measurements are correct, then Eqs.(12), (13), (14) are

$$\mathbf{AX} = \mathbf{L} \tag{17}$$

where $\mathbf{X} = (dx_0, dy_0, dz_0, d\theta_0)^T$. Resulting in the error equation

$$\mathbf{V} = \mathbf{L} - \mathbf{A}\mathbf{X} \tag{18}$$

Furthermore the error variance of the observed value \mathbf{L} can be derived from the averages of the square of Eqs.(12), (13), and (14) as

$$\boldsymbol{\Sigma}_{L} = \mathbf{K}_{1} \boldsymbol{\Sigma} \mathbf{K}_{1}^{T} + \mathbf{K}_{2} \boldsymbol{\Sigma}_{p} \mathbf{K}_{2}^{T}$$
(19)

where \mathbf{K}_1 and \mathbf{K}_2 are

And Σ is the error variance matrix for the positions of robot 1 and 2 given by Eqs. (26) and (27)

at the previous movement, and Σ_p is the error variance matrix for the measurement of distances, azimuth and elevation angles that are the intrinsic values of the measurement devices.

$$\boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12} \\ \boldsymbol{\Sigma}_{21} & \boldsymbol{\Sigma}_{22} \end{pmatrix}$$
(22)

$$\boldsymbol{\Sigma}_p = diag(\sigma_r^2, \sigma_\phi^2, \sigma_\psi^2) \tag{23}$$

From Eqs.(18) and (19) the change in position of robot 0 (**X**) that minimizes the sum of the squared residual error under the weight of Σ_L^{-1} can be derived by solving the following equation

$$\min \mathbf{V}^T \mathbf{\Sigma}_L^{-1} \mathbf{V} \tag{24}$$

 \mathbf{as}

$$\mathbf{X} = (\mathbf{A}^T \boldsymbol{\Sigma}_L^{-1} \mathbf{A})^{-1} \mathbf{A}^T \boldsymbol{\Sigma}_L^{-1} \mathbf{L}$$

= **BL** (25)

where, $\mathbf{B} = (\mathbf{A}^T \boldsymbol{\Sigma}_L^{-1} \mathbf{A})^{-1} \mathbf{A}^T \boldsymbol{\Sigma}_L^{-1}$.

Finally, the following steps make it possible to calculate the optimum position of robot 0, i) assume arbitrary position of robot 0 as $\tilde{\mathbf{P}}_0$. ii) calculate $\mathbf{X} = (dx_0, dy_0, dz_0, d\theta_0)^T$ form Eq.(25). iii) repeat $\tilde{\mathbf{P}}_0 \leftarrow \tilde{\mathbf{P}}_0 + \mathbf{X}$ until $\mathbf{X} \to 0$. And thus the error variance of the position of robot 0 can be calculated from Eq.(25) as

$$\boldsymbol{\Sigma}_{00} = \mathbf{B}\boldsymbol{\Sigma}_L \mathbf{B}^T = (\mathbf{A}^T \boldsymbol{\Sigma}_L^{-1} \mathbf{A})^{-1} \qquad (26)$$

and the covariance matrixes between robots 0 and 1, and robots 0 and 2 are

$$(\boldsymbol{\Sigma}_{01}, \boldsymbol{\Sigma}_{02}) = \mathbf{B}\mathbf{K}_1\boldsymbol{\Sigma}$$
(27)

By repeating the above steps, the positioning accuracy of the robot after performing several measurements successively can be estimated by calculating the error variance matrix and covariance matrixes from Eqs.(26) and (27).

4. Move and Measurement Experiment with CPS-II

This section reports the results of position identification experiments with CPS-II in cases two child robots and three redundant child robots are used.

4.1. Move and Measurement Experiment

An experiment for measuring positioning accuracy was performed using our prototype mobile robot system (CPS-II) in an indoor environment. In this experiment, three robots traveled on an even surface along the wall in a room measuring 6 m x 12 m. The robots traveled around the room clockwise while repeatedly determining their own position using CPS. When they returned to almost the same area of the initial positions, the position of the parent robot was compared with its initial position determined by fixed-point measurement.

Fig. 9 shows how the robots moved in this experiment. In this figure, the parent robots are each represented by a square and the child robot is represented by a circle. The three robots started from their original positions (indicated by an upside-down triangle at the lower left) along the y axis. They repeated the operation cycle shown in Fig. 8 ten times to return to their initial positions. The route traveled by the parent robot is indicated with a solid line in the figure. Table 2 lists the average errors of position and attitude of the parent robot. The total travel distance of the parent robot was 21.5 m. The average positioning error was 59.2 mm (0.28% of the total travel distance); the average attitude error was 0.27 degree.

For comparison, a robot with an encoder of 47,520 pulses per rotation attached to its driving wheel was simply moved in a straight-line for a distance of 5 m and positioning accuracy by dead reckoning was measured. The positioning accuracy was 1.4% of the total travel distance. Next, the robot traveled in the same environment of Fig. 9 with dead reckoning only. The route traveled by the robot is indicated with a dotted line in Fig. 9. When a change of direction was involved, azimuth errors greatly affected the positioning accuracy, pushing the error rate up to as much as 10%. This proves that CPS can determine position with a much higher precision than dead reckoning can.

This experiment was performed on an even surface with no roughness. The positioning accuracy of CPS, however, is not affected by an environment where CPS is used. It is expected that the positioning accuracy obtained by this experiment could also be obtained easily for an uneven outdoor terrain, and obtained accuracy of CPS can be far better than that of dead reckoning.

4.2. Fusion of redundant positioning information

Next, we perform a moving experiment using one parent robot and two and three child robots, and make a comparison of the positioning accuracy with and without redundant robots.

In this experiment, one parent robot and three child robots move in the same environment as the previous experiment while performing position identification by CPS, and the positioning accuracies in cases when only three and four robots are used for parent robot positioning are compared after robots travel around the room.

Fig. 10 shows how the robots moved in this experiment and Fig. 11 shows moving procedure of each robot. In Fig. 10, parent robots are each represented by a square and the child robot is represented by a circle. They repeated the operation



Fig. 9. An example of the experimental results with 3 robots.

Table 2. Position and attitude accuracy of CPS-II after robot 0 moves a distance of 21.5 m $\,$



cycle shown in Fig. 11 ten times to return to their initial positions. The route traveled by the parent robot is indicated with a solid line in the figure. Table 3 lists the average errors of position and attitude of the parent robot for following the cases.



Fig. 10. An example of the experimental results with 4 robots.



Fig. 11. Moving strategy in each measurement cycle.

- 1. Three robots are used.
- 2. Four robots are used and the position of parent robot is calculated using weighted least squares method proposed in Section 3.
- 3. Four robots are used and the position of parent robot is calculated as the arithmetic mean of three positions obtained from the set of robot 0-1-2, robot 0-1-3, and robot 0-2-3, respectively.

This table shows that positioning principle that we proposed in Section 3 can realize lower accumulation of positioning error than that in other cases, and make it possible to perform the appropriate integration of redundant position information.

5. Conclusion

We have proposed the cooperative positioning system (CPS) that can accurately determine the positions of multiple robots through cooperative control of individual robots in the group. This technique can accurately position mobile robots working in uncharted, off-road, or otherwise difficult to maneuver environments for which accurate positioning would be impossible with conventional techniques. This method is therefore promising for many fields. Next, we outlined a second prototype CPS machine model (CPS-II) and discussed the positioning principle using the variance of positioning error and weighted least squares method. Position identification experiments show that CPS could perform far more accurate measurements than the dead reckoning method even in uneven

Table 3. Positioning and attitude accuracy of CPS-II with 3 and 4 robots for the motion of Fig. 10. Positioning error ratios are also shown.

	Position	Attitude
3 robots	67.9mm (0.32%)	0.43 deg.
4 robots Proposed method	$26.3 \mathrm{mm}(0.12\%)$	0.32 deg.
4 robots Arithmetic means	$53.0 \mathrm{mm}(0.25\%)$	0.83 deg.

environments and positioning accuracy could improve with increasing the number of robots.

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