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Grasp Stability Evaluation based on Energy Tolerance in Potential Field

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***Abstract*— We propose an evaluation method of grasp stability which takes into account the elastic deformation of fingertips from the viewpoint of energy. An evaluation value of grasp stability is derived as the minimum energy which causes slippage of a fingertip on its contact surface. To formulate the evaluation value, the elastic potential energy of fingertips and the gravitational potential energy of a grasped object are considered. It is ensured that fingertips do not slip on grasped object surfaces if the external energy applied to the object is less than the evaluation value. Since our evaluation value explicitly considers the deformation values of fingertips, grasp stability is evaluated by taking into consideration the contact forces generated by the deformation. The effectiveness of our method is verified through numerical examples.**

I. INTRODUCTION

Grasp stability is a widely used concept for multi-fingered hands to prevent grasped objects from dropping. Various evaluation methods of grasp stability have been proposed so far [1]. Among them, the methods based on force closure [2] are widely used. These methods evaluate grasp stability by using the minimum norm of an external force/moment which may cause slippage of a fingertip on its contact surface.

This paper newly proposes an evaluation method of grasp stability for multi-fingered hands which have soft fingertips. Our method takes into account the effect of fingertip elasticity, and evaluates grasp stability by using the minimum energy which may cause slippage of a fingertip on its contact surface. The force closure evaluates grasp stability from the viewpoint of force/moment, whereas our method evaluates grasp stability from the viewpoint of energy. Our method is not the one just adding the effect of the fingertip elasticity to conventional methods but the one which has the following significant improvement from conventional methods.

In the definition of the force closure, the total force/moment set generated for a grasped object is calculated by using the Minkowski sum of the edge vectors spanning an approximated friction cone at each contact point. This implicitly assumes that each fingertip can exert an arbitrary contact force within the friction cone constraint. However, in actual grasp system, contact forces are controlled by given control law and generated by the deformation of fingertips. The set of the actual contact forces are not the same as the set considered in force closure condition. Therefore the definition of force closure can not be used for evaluating for the given values of contact forces. In the proposed method, the deformation of the fingertips is taken into account for the evaluation.

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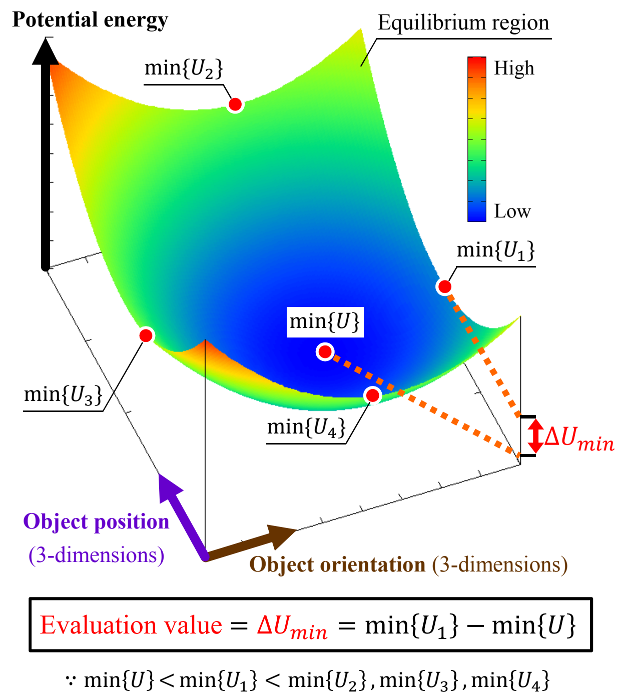


Fig. 1: Conceptual figure of a potential field : Evaluation value of grasp stability is the minimum energy which may cause slippage of a fingertip on its contact surface.

In the definition of the force closure, an external force/moment applied to a grasped object is compensated by joint torque of each finger. In the proposed definition of grasp stability, an external force/moment applied to a grasped object is compensated by elasticity at each fingertip. Hence, we can regard that these two definitions are complementary to each other when a multi-fingered hand grasps an object with elasticity at each fingertip.

In the previous paper [3], we proposed a basic part of the method with linear springs, a point contact model, and a few numerical examples. Different from the previous paper, this paper formulates the elastic energy taking the full six-dimensional deformation of the fingertips into account by modeling the surface contact at each contact point. We derive an evaluation value of grasp stability by considering the relationship between the position/orientation of a grasped object and the potential energy of a grasp system. The potential energy of a grasp system is calculated by summing up the elastic potential energy of each fingertip and the gravitational potential energy of a grasped object.

II. RELATED WORKS

After Ferrari and Canny [4] proposed an evaluation method of grasp stability based on the force closure, several researchers have extended this method. Ciocarlie et al. [5] took into account the static friction torque. Harada et al. [6] and Tsuji et al. [7] took into account the shape of contact areas. These methods cannot be used for evaluating the grasp stability for given values of contact force generated by the deformation of fingertips.

Several evaluation methods of grasp stability from the viewpoint of potential energy have been proposed [8]-[13]. In these methods, the curvature of the potential energy of the grasp system is evaluated. Cutkosky and Kao [10], Maekawa et al. [11] took into account the joint compliance of fingers. Choi et al. [12] and Yamada et al. [13] took into account the curvature of contact surfaces. In these methods, it is not explicitly ensured that each contact force satisfies the friction cone constraint.

Several researchers analyzed the effect of soft fingertips. Hanafusa and Asada [14][15] proved that the stable grasp is realized when potential energy of the grasp system is a local minimum. Tahara et al. [16] proved that a grasp system satisfies the passivity by the effect of viscosity of fingertips and finger joint. In view of these knowledge, to utilize soft fingertips is effective in grasping an object stably. Inoue and Hirai [17], [18] model soft fingertips and analyse stability. However, there has been no research of the quantitative evaluation of grasp stability where the effect of fingertip elasticity and friction constraint are taken into account.

III. OVERVIEW OF OUR METHOD

We numerically evaluate the grasp stability of an object being grasped by using soft fingertips under gravity. We start with several assumptions as follows: The grasped object is rigid, and its shape and mass are known. The contact surfaces of an object are planar, and their normal direction is known. The fingertips have a hemispherical shape, and their position/orientation do not change during grasping. This also means that finger joint angles do not change during grasping.

We define the initial grasping state such that the following conditions are satisfied: Each fingertip deforms only in the normal direction of its contact surface, and their deformation values are given. The bottom surface as shown in Fig. 2 of each fingertip is parallel to its contact surface.

A. Calculation of Evaluation Value

In order to calculate an evaluation value of grasp stability, we construct a potential field given a grasping posture. Fig. 1 shows a conceptual figure of a potential field. A potential field denotes the relationship between the object position/orientation and the potential energy of a grasp system. Though object position/orientation has 6 dimensions, it is expressed in 2 dimensions in Fig. 1. The potential energy of a grasp system is obtained by summing up the elastic potential energy of each fingertip and the gravitational potential energy of the object. The detail of the constructing method of a potential field is described in Section V.

Let a equilibrium region denote the region in a potential field where no fingertip slips. A fingertip begins to slip on the boundary of the equilibrium region. When object position/orientation is in the equilibrium region, stable grasping can be kept. Here, let $\min\{U\}$ be the minimum potential energy in the equilibrium region, let $\min\{U_i\}$ be the minimum potential energy on i-th boundary of the equilibrium region, and let ΔU_{min} be the minimum difference between $\min\{U\}$ and $\min\{U_i\}$ as follows:

$$\Delta U_{min} = \min\{\min\{U_i\} - \min\{U\}, i = 1, \cdots, N_b\}, \quad (1)$$

where N_b is the number of the boundaries. When energy ΔU_{min} is applied to the object, object position/orientation may reach a boundary of the equilibrium region. Then a fingertip may begin to slip on its contact surface. Hence, this research evaluates the grasp stability by using the ΔU_{min}.

B. Stable Grasp Constraint

In Fig. 1, the evaluation value of grasp stability is larger than 0. In this case, no fingertip begins to slip unless energy is inputted to the grasp system. Hence, the grasp posture is found out to be stable However, if $\min\{U\}$ is placed on a boundary of the equilibrium region, a fingertip begins to slip. In this case, the evaluation value of grasp stability is 0. Hence, the grasp posture is found out to be unstable.

Since our method utilizes energy as an evaluation index, it is possible to derive a stable grasp constraint even when the object has the kinetic energy. By evaluating grasp stability taking into account the kinetic energy, it is possible to utilize our method to the case where the grasped object is dynamically moving such as object transportation task. Let K and U be the kinetic energy and the potential energy respectively. If the following inequality is satisfied, stable grasp is ensured:

$$U + K < \min\{U\} + \Delta U_{min}. \quad (2)$$

As time passes, kinetic energy of the object is dispersed by the viscosity of the fingertips, and the object will stop at the equilibrium point.

IV. FINGERTIP DEFORMATION MODEL AND FRICTION CONSTRAINT

A. Fingertip Deformation Model

When object position/orientation changes from the initial grasping state, the bottom surface of each fingertip is not always parallel to its contact surface, and each fingertip is deformed also in the tangential and torsional direction of its contact surface. The elastic potential energy is formulated as independent of the initial deformation value of the fingertip.

In this paper, we utilize the parallel-distributed model [17], [18] as a soft fingertip model. The derivation of the following formulation is described in Appendix.

Deformation in the Normal Direction : Let d_{ni} be the deformation value in the normal direction of the i-th fingertip, and let ϕ_i be the relative angle of the i-th fingertip bottom surface and its contact surface. Elastic potential energy U_{ni} is given as follows:

$$U_{ni} = \frac{\pi E d_{ni}^3}{3\cos^2\phi_i}, \tag{3}$$

where E is Young's modulus of fingertips material. Elastic force f_{ni} is given as follows:

$$f_{ni} = \frac{\partial U_{ni}}{\partial d_{ni}} = \frac{\pi E d_{ni}^2}{\cos^2\phi_i}. \tag{4}$$

Deformation in the Tangential Direction : Let d_{ti} be the deformation value in the tangential direction of the i-th fingertip. Elastic potential energy U_{ti} is given as follows:

$$U_{ti} = \pi E d_{ni} d_{ti}^2. \tag{5}$$

Elastic force f_{ti} is given as follows:

$$f_{ti} = \frac{\partial U_{ti}}{\partial d_{ti}} = 2\pi E d_{ni} d_{ti}. \tag{6}$$

Deformation in the Torsional Direction : Let $\theta_{\tau i}$ be the deformation value in the torsional direction of the i-th fingertip. Elastic potential energy $U_{\tau i}$ is given as follows:

$$U_{\tau i} = \pi E d_{ni}^2 \left(r - \frac{1}{3}d_{ni}\right)\theta_{\tau i}^2, \tag{7}$$

where r is the radius of the fingertips. Elastic moment τ_{ni} is given as follows:

$$\tau_{ni} = \frac{\partial U_{\tau i}}{\partial \theta_{\tau i}} = 2\pi E d_{ni}^2 \left(r - \frac{1}{3}d_{ni}\right)\theta_{ni}. \tag{8}$$

B. Friction Constraint

We utilize a friction constraint proposed by Howe and Cutkosky [19]. If the following inequality is satisfied, the i-th fingertip does not slip on its contact surface:

$$f_{ti}^2 + \frac{\tau_{ni}^2}{e_{ni}^2} \leq \mu^2 f_{ni}^2, \tag{9}$$

where μ is the static friction coefficient, and e_{ni} is an eccentricity parameter which depends on the shape of the contact area.

e_{ni} is derived by using Winkler elastic foundation [20] as pressure distribution model. If the contact region is elliptical shape, we have $e_{ni} = \frac{8}{15}\mu\sqrt{ab}$ where a and b are semi-axes. Here, since the shape of contact regions of our research is circular, e_{ni} is given as follows:

$$e_{ni} = \frac{8}{15}\mu\sqrt{r^2 - (r - d_{ni})^2}. \tag{10}$$

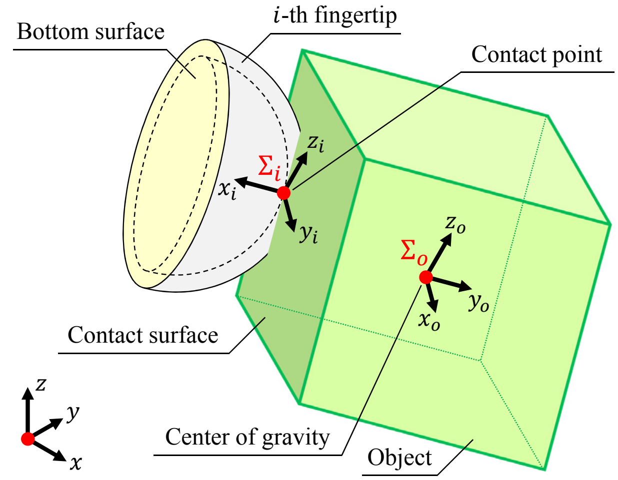


Fig. 2: Coordinate systems : Σ_o denotes the object coordinate system, and Σ_i denotes the contact point coordinate system of the i-th fingertip.

V. CONSTRUCTION OF POTENTIAL FIELD

We formulate the relationship between the object position/orientation and the potential energy of a grasp system. First, we calculate the deformation value of each fingertip of the object position/orientation. Next, by using this result, we calculate the elastic potential energy of each fingertip. In addition, we calculate the gravitational potential energy of the object at each object position. By summing these energy terms, we calculate the potential energy of the grasp system.

We define local coordinate systems Σ_o and Σ_i as shown in Fig. 2 where Σ_o denotes the object coordinate system with its origin attached at the gravity center of the object. Let $\boldsymbol{p}_o \in \mathbb{R}^3$ and $\boldsymbol{R}_o \in \mathrm{SO}(3)$ be the position vector and the rotation matrix of Σ_o, with respect to the inertial coordinate system. Σ_i denotes the contact point coordinate system of the i-th fingertip with its origin attached at the i-th contact point (the center of the contact region). In the initial grasping state, let ${}^o\boldsymbol{p}_i \in \mathbb{R}^3$ and ${}^o\boldsymbol{R}_i \in \mathrm{SO}(3)$ be the the position vector and the rotation matrix of Σ_i, with respect to Σ_o. In addition, let the positive direction of x axis of Σ_i be the normal direction of the i-th contact surface.

A. Calculation of Fingertips Deformation Value from Object Position and Orientation

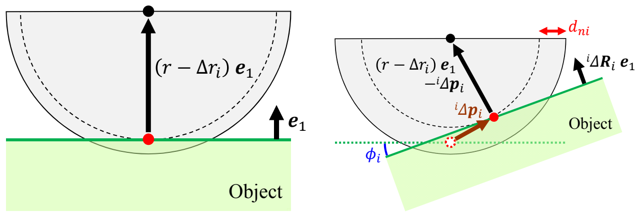
Let $\Delta\boldsymbol{p} \in \mathbb{R}^3$ and $\Delta\boldsymbol{R} \in \mathrm{SO}(3)$ be the change of object position/orientation from the initial grasping state, with respect to the inertial coordinate system. Let ${}^i\Delta\boldsymbol{p}_i \in \mathbb{R}^3$ and ${}^i\Delta\boldsymbol{R}_i \in \mathrm{SO}(3)$ be the change of position/orientation of Σ_i, with respect to Σ_i. They are given as follows:

$${}^i\Delta\boldsymbol{p}_i = \boldsymbol{R}_i^{\mathrm{T}}\left\{(\Delta\boldsymbol{R} - \boldsymbol{I})\left(\boldsymbol{R}_o{}^o\boldsymbol{p}_i + \boldsymbol{p}_o\right) + \Delta\boldsymbol{p}\right\}, \tag{11}$$

$${}^i\Delta\boldsymbol{R}_i = \boldsymbol{R}_i^{\mathrm{T}}\Delta\boldsymbol{R}\boldsymbol{R}_i\,, \tag{12}$$

where $\boldsymbol{R}_i = \boldsymbol{R}_o{}^o\boldsymbol{R}_i$. We calculate deformation value of each fingertip of ${}^i\Delta\boldsymbol{p}_i$ and ${}^i\Delta\boldsymbol{R}_i$.

Deformation in the Normal Direction : As shown in Fig. 3, from the geometrical relationship, d_{ni} and ϕ_i is given



(a) Initial grasping state: The deformation value is Δr_i.

(b) After position/orientation change: The deformation value is d_{ni}, and the relative angle is ϕ_i.

Fig. 3: Fingertip deformation in the normal direction

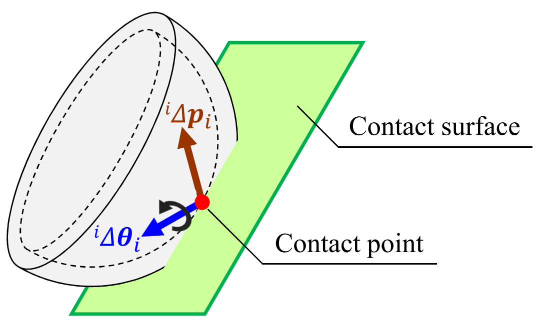


Fig. 4: Position/orientation change of Σ_i : ${}^i\Delta \boldsymbol{p}_i$ denotes the displacement of the i-th contact point, and ${}^i\Delta \boldsymbol{\theta}_i \in \mathbb{R}^3$ is a rotation vector.

as follows:

$$d_{ni} = r - \left\{ (r - \Delta r_i)\, \boldsymbol{e}_1 - {}^i\Delta \boldsymbol{p}_i \right\}^{\mathrm{T}} \left({}^i\Delta \boldsymbol{R}_i\, \boldsymbol{e}_1 \right), \tag{13}$$

$$\phi_i = \cos^{-1} \left\{ \boldsymbol{e}_1{}^{\mathrm{T}} \left({}^i\Delta \boldsymbol{R}_i\, \boldsymbol{e}_1 \right) \right\}, \tag{14}$$

where Δr_i is the initial deformation value of the i-th fingertip, and $\boldsymbol{e}_1 = (1, 0, 0)^{\mathrm{T}}$.

Deformation in the Tangential Direction : As shown in Fig. 4, ${}^i\Delta \boldsymbol{p}_i$ denotes the displacement of the i-th contact point. We calculate deformation value d_{ti} as the tangential component of ${}^i\Delta \boldsymbol{p}_i$ as follows:

$$d_{ti} = \| \boldsymbol{D}_{23}\, {}^i\Delta \boldsymbol{p}_i \|, \tag{15}$$

where $\boldsymbol{D}_{23} = \mathrm{diag}(0, 1, 1)$.

Deformation in the Torsional Direction : We convert the rotation matrix ${}^i\Delta \boldsymbol{R}_i$ into a rotation vector ${}^i\Delta \boldsymbol{\theta}_i \in \mathbb{R}^3$, as shown in Fig. 4. $\widehat{{}^i\Delta \boldsymbol{\theta}_i}$ denotes equivalent rotation axis, and $\|{}^i\Delta \boldsymbol{\theta}_i\|$ denotes equivalent rotation angle. We calculate deformation value $\theta_{\tau i}$ as the normal component of ${}^i\Delta \boldsymbol{\theta}_i$ as follows:

$$\theta_{\tau i} = {}^i\Delta \boldsymbol{\theta}_i{}^{\mathrm{T}} \boldsymbol{e}_1. \tag{16}$$

B. Calculation of Potential Energy of Grasp System

Elastic potential energy of the i-th fingertip U_{ei} is given as follows:

$$U_{ei} = U_{ni} + U_{ti} + U_{\tau i}. \tag{17}$$

Let zero point for gravitational potential energy be the initial grasping state. The gravitational potential energy of the object U_g is given as follows:

$$U_g = mg \Delta \boldsymbol{p}^{\mathrm{T}} \boldsymbol{e}_3 , \tag{18}$$

where m is the mass of the object, g is the gravitational acceleration, and $\boldsymbol{e}_3 = (0, 0, 1)^{\mathrm{T}}$.

The potential energy of the grasp system U is given as follows:

$$U = \sum_{i=1}^{N_f} U_{ei} + U_g , \tag{19}$$

where N_f is the number of the fingertips.

C. Boundary Conditions of Equilibrium Region

As described in Section III, a boundary of the equilibrium region of a potential field denotes a state in which a fingertip begins to slip. From the friction constraint described in (9), the following boundary conditions of the equilibrium region are given:

$$f_{ti}^2 + \frac{\tau_{ni}^2}{e_{ni}^2} = \mu^2 f_{ni}^2 . \tag{20}$$

Since each finger has the upper limit of the contact force, the following boundary conditions of the equilibrium region are given:

$$f_{ni} = f_{max} , \tag{21}$$

where f_{max} is the upper limit of the contact force in the normal direction.

VI. CALCULATION OF THE MINIMUM POTENTIAL ENERGY

The minimum potential energy in the equilibrium region of a potential field is obtained by solving the following nonlinear programming problem:

$$\begin{aligned} &\text{minimize} \quad U \\ &\text{subject to} \quad \frac{d_{ni}}{\cos \phi_i} \le \sqrt{\frac{f_{max}}{\pi E}} \qquad (i = 1, \cdots, N_f) \\ &\qquad\qquad\quad f_{ti}^2 + \frac{\tau_{ni}^2}{e_{ni}^2} \le \mu^2 f_{ni}^2 \qquad (i = 1, \cdots, N_f) . \end{aligned}$$

The first constraint denotes that the contact force in the normal direction f_{ni} is less than its upper limit f_{max}. The second constraint denotes that the i-th fingertip does not slip.

In order to calculate the minimum potential energy on a boundary, it is necessary to add a constraint. If the target boundary denotes a state in which the j-th fingertip begins to slip, the minimum potential energy is obtained by solving the following nonlinear programming problem:

$$\begin{aligned} &\text{minimize} \quad U \\ &\text{subject to} \quad f_{tj}^2 + \frac{\tau_{nj}^2}{e_{nj}^2} = \mu^2 f_{nj}^2 \\ &\qquad\qquad\quad f_{ti}^2 + \frac{\tau_{ni}^2}{e_{ni}^2} \le \mu^2 f_{ni}^2 \qquad (i = 1, \cdots, N_f) \\ &\qquad\qquad\quad \frac{d_{ni}}{\cos \phi_i} \le \sqrt{\frac{f_{max}}{\pi E}} \qquad (i = 1, \cdots, N_f) . \end{aligned}$$

If the target boundary denotes a state in which the j-th fingertip exceeds the upper limit of the contact force, the minimum potential energy is obtained by solving the following nonlinear programming problem:

$$\begin{aligned} &\text{minimize} \quad U \\ &\text{subject to} \quad \frac{d_{nj}}{\cos\phi_j} = \sqrt{\frac{f_{max}}{\pi E}} \\ &\qquad f_{ti}^2 + \frac{\tau_{ni}^2}{e_{ni}^2} \le \mu^2 f_{ni}^2 \qquad (i = 1, \cdots, N_f) \\ &\qquad \frac{d_{ni}}{\cos\phi_i} \le \sqrt{\frac{f_{max}}{\pi E}} \qquad (i = 1, \cdots, N_f). \end{aligned}$$

VII. NUMERICAL EXAMPLES

We demonstrate the effectiveness of our method by numerical examples. In all cases, we set $\boldsymbol{p}_o = \boldsymbol{0}$, $\boldsymbol{R}_o = \boldsymbol{I}$, $r = 25\,(\mathrm{mm})$, $\mu = 0.5$, and $f_{max} = 15\,(\mathrm{N})$. We utilize Ipopt [21] and ADOL-C [22] to solve nonlinear programming problems.

We investigate the relationship between fingertip elasticity, initial position, and grasp stability. Here, we consider a situation in which a cube is being grasped by using 2 fingertips, as shown in Fig. 5a. Each side length of the cube is $100\,(\mathrm{mm})$, and each contact point position is the center of its contact surface. The object mass is set as $m = 0.0\,(\mathrm{kg})$ in this experiment. In addition, we change the Young's modulus of each fingertip E from $2 \times 10^5\,(\mathrm{Pa})$ to $1 \times 10^6\,(\mathrm{Pa})$, and change the initial deformation value of each fingertip Δr_i from $1\,(\mathrm{mm})$ to $5\,(\mathrm{mm})$.

Fig. 6 shows the evaluation value for each E and Δr_i. When Δr_i increases from $1\,(\mathrm{mm})$, the evaluation value starts to increase. It means that slippage of the fingertips is harder to occur as initial deformation increases. However, the evaluation value starts to decrease on the way, since the contact force in the normal direction f_{ni} is easy to exceed its upper limit f_{max}. This result shows that our method can determine optimal value of initial deformation, taking into account the elasticity of fingertip.

Fig. 7a shows the maximum evaluation value for each E. When E increases, then the evaluation value decreases. This result shows that the effectiveness of softer fingertips are quantitatively evaluated.

As another example, we consider a situation in which a triangular prism is being grasped by using 3 fingertips, as shown in Fig. 5b. Each side length of the triangular prism is $100\,(\mathrm{mm})$, and each contact point position is the center of its contact surface. We calculated evaluation values in the same way as the previous example, and obtained a similar result. Fig. 7b shows the maximum evaluation value for each E. When the object mass is larger, the evaluation value decreases. It is useful that the mass of the object can be considered in real application.

VIII. CONCLUSION

In this paper, we propose an evaluation method of grasp stability which takes into account the effect of fingertip elasticity. In our method, the evaluation value of grasp stability is the minimum energy which causes slippage of a fingertip on its contact surface.

First, we define the fingertip deformation model and frictional constraint. Second, we described a construction method of the potential field. Third, we described a calculation method of the minimum potential energy. Finally, through numerical examples, it was shown that our method can evaluate grasp stability, taking into account the fingertip elasticity and the magnitude of the internal force.

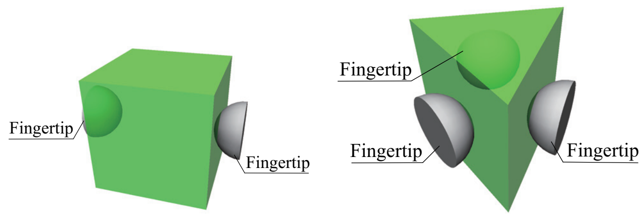


Fig. 5: Grasp posture for cube and triangular prism

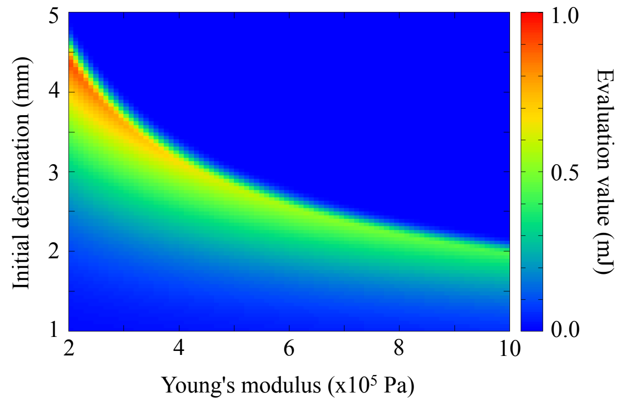


Fig. 6: Evaluation value for each Young's module and initial deformation

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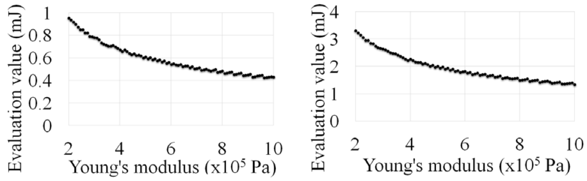
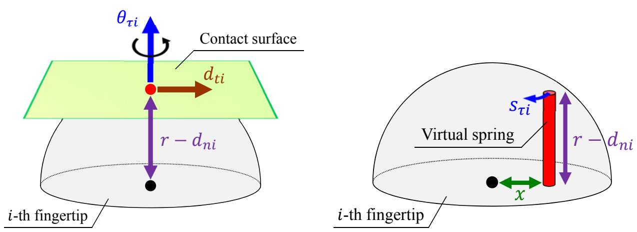


(a) cube (b) triangular prism

Fig. 7: The maximum evaluation value for each Young's module



(a) The tangential/torsional deformation.

(b) Deformation of a virtual spring.

Fig. 8: Fingertip deformation in the normal direction

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Appendix

Parallel-distributed Model of Soft Fingertips

In the parallel-distributed model [17][18], fingertips are treated as if they were composed of an infinite number of virtual linear springs standing vertically. We formulate the deformation in the normal/tangential/torsional direction as there is no deformation order.

Deformation in the Normal Direction : Equation (3) was derived in [17]. Elastic force f_{ni} is obtained by doing partial differentiation as shown in (4).

Deformation in the Tangential Direction : The following equations was derived in [18]:

$$U_{tangent} = \pi E(d_n^2 d_t \tan\theta_p + d_n d_t^2), \tag{22}$$

$$U_{side} = \pi E d_n d_s^2. \tag{23}$$

Since there is no deformation order, as shown in Fig. 8a, we consider that the tangential deformation occurs when $\theta_p = 0$. In this case, the following equation is obtained:

$$U_{tangent} + U_{side} = \pi E d_n (d_t^2 + d_s^2). \tag{24}$$

Because $d_t^2 + d_s^2$ is the square of the tangential deformation value, (5) is obtained. Elastic force f_{ti} is obtained by doing partial differentiation as shown in (6).

Deformation in the Torsional Direction : As shown in Fig. 8b, we consider a virtual spring that the distance from the fingertip center is x. The spring constant of the spring k_i is given as follows:

$$k_i = \frac{EdS}{\sqrt{r^2 - x^2}}\,, \tag{25}$$

where dS is the sectional area of the spring. The deformation value of the spring $s_{\tau i}$ is given as follows:

$$s_{\tau i} = x\theta_{\tau i}. \tag{26}$$

The elastic potential energy $U_{\tau i}$ is given as follows:

$$U_{\tau i} = \iint_{cr} \frac{1}{2} k_i s_{\tau i}^2 = \iint_{cr} \frac{E\theta_{\tau i}^2 x^2}{2\sqrt{r^2 - x^2}} dS\,, \tag{27}$$

where cr denotes the contact region between the i-th fingertip and the object. Since the shape of the contact region is circular, this integration is calculated as follows:

$$U_{\tau i} = \int_0^{\sqrt{r^2-(r-d_{ni})^2}} \int_0^{2\pi} \frac{E\theta_{\tau i}^2 x^3}{2\sqrt{r^2 - x^2}} d\varphi dx. \tag{28}$$

By calculating this integration, (7) is obtained. Elastic moment τ_{ni} is obtained by doing partial differentiation as shown in (8).