

Tissue Surface Model Mapping onto Arbitrary Target Surface Based on Self-organizing Deformable Model

Shoko Miyauchi*, Ken'ichi Morooka*, Yasushi Miyagi[†], Takaichi Fukuda[‡], Tokuo Tsuji* and Ryo Kurazume*

* Graduate School of Information Science and Electrical Engineering, Kyushu University
Motooka 744, Nishi-ku, Fukuoka 819-0395 Japan

Email: miyauchi@irvs.ait.kyushu-u.ac.jp, morooka@ait.kyushu-u.ac.jp

[†] Department of Neurosurgery, Kaizuka Hospital, Fukuoka 812-0053 Japan

[‡] Graduate School of Medical Sciences, Kumamoto University, Kumamoto 860-8556 Japan

Abstract—This paper proposes a new method for mapping a tissue surface model onto an arbitrary target surface while preserving the geometrical features of the tissue surface. In our method, firstly, the tissue model is roughly deformed by using Self-organizing Deformable Model. Since the deformed model may contain folded patches, the folded patches are removed. Moreover, by Free-Form Deformation (FFD), and the area- and angle-preserving mapping, the model is mapped onto the target surface while preserving geometrical properties of the original model. From several experimental results, we can conclude that the proposed method can map tissue models onto arbitrary target surface without foldovers.

I. INTRODUCTION

Recent medical imaging devices can provide high resolution medical images. Tissue models generated by the images are used in support systems for diagnosis and treatment [1]. One technique using the models is to find the relationship and similarity between the models of a target tissue. Each tissue has fairly consistent shape while the shape varies across individuals. By analyzing many shape patterns of the tissue, a statistical shape model (SSM) is build to identify the considerable natural variability of the tissue. Because of the shape prior information which SSM provides, recent works show that SSM-based techniques have obtained considerable success in the tissue detection from medical images [2].

The fundamental process for building SSM is to establish the correspondence between the models. Generally, triangular mesh models of the tissue have different number of vertices and different topology. The correspondence problem becomes complex in the case of the tissue with complex shape such as the human brain. One solution for this problem is to map the models onto a target surface with simple shape including a plane and a sphere. Since the target surface is described by a simple parametric function, such mapping-based approach allows to easily determine the correspondences on the target surface through the parameters in the function.

To achieve this, the mapping method needs to guarantee one-to-one correspondence between the model and the target surface while preserving the original geometric properties as far as possible. To compare the models effectively, it is desirable to control the mapping so that anatomical features involved in the model are constrained to lie at specific locations on the target surface. Moreover, if the mapping deals with arbitrary shape, the effective mapping of tissue models

is realized by selecting suitable target surface according to the tissue shape. However, there are few mapping methods satisfying the three requirements.

In this paper, we propose a new method for mapping a tissue surface model onto an arbitrary target surface while preserving the geometrical features of the original tissue model. The proposed method uses Self-organizing Deformable Model (SDM) [3], [4], which is a deformable model guided by competitive learning and an energy minimization approach. The SDM allows to move some vertices included in the SDM toward specific points on the target surface and to choose arbitrary shapes for both the SDM and target surface.

On the contrary, when the SDM is applied to the brain surface model, multiple patches in the model may be projected onto the limited area of the target surface. When the model contain such overlap, called foldover, the mapping between the model and the target surface is not bijective. To avoid the foldover, the original SDM algorithm [3] introduces certain techniques, however, does not necessarily guarantee foldover-free mapping. Matsui et al. [4] incorporated Least-squares Meshes (LSM) into the SDM algorithm. In the LSM method, a matrix is employed to represent the point correspondence between the model and the target surface. The model is deformed by the inverse matrix. Since the matrix consists of all vertices in the model, the computation of the inverse matrix increases exponentially when the model has a large number of vertices. Moreover, to perform LSM, a user needs to specify point correspondences between the model and the target surface. In case of the brain surface with complex shape, many point correspondences are needed to avoid the foldovers.

To solve the problems, in our method, the tissue model is roughly deformed by using the original SDM deformation, and folded patches are removed from the deformed model. Moreover, by Free-Form Deformation (FFD), and the area- and angle-preserving mapping, the model is mapped onto the target surface while preserving geometrical properties of the original model.

II. TISSUE MODEL MAPPING BASED ON SDM

This section explains the detail of the proposed mapping method. Here, we describe several definitions used in our method. Given a vertex v_i , the 1-ring region W_i of v_i is defined by a set of the patches $w_k^{(i)}$ ($k = 1, 2, \dots, N_p^{(i)}$) which have

the vertex v_i . $N_p^{(i)}$ denotes the number of patches contained in the 1-ring region.

A. Self-organizing Deformable Model

The SDM is a deformable mesh model represented by triangular patches. A target surface is represented by a set of points on the surface, called control points. Arbitrary shapes can be chosen for both the SDM and target surface, and the size of a target surface is sufficient to cover the initial SDM, provided that the SDM and the target surface have the same topological type. Feature vertices are the vertices which correspond to specific locations on the target surface, and the control points close to the locations are the corresponding points of the feature vertices. In the SDM algorithm, a given mesh model is deformed to fit to the target surface by competitive learning and minimizing an energy function. See the ref [3] in detail of the SDM algorithm.

B. Foldover removal

When a brain surface model is mapped onto a given target surface by using the original SDM algorithm, the deformed brain model may contain a foldover. The foldover is removed based on the method proposed by Athanasiadis et al. [5]. Since the method in [5] dealt with only a spherical surface as a target surface. We extend the method in [5] to apply to target surface with arbitrary shape.

The foldover occurs when a vertex is not included in its 1-ring region. Considering this, the foldover is removed by moving the vertices to within their 1-ring region. This movement is made by moving each vertex v_i forward the location computed by

$$\mathbf{v}_i = \phi \left(\frac{\sum_k A_{w_k}^{(i)} \mathbf{C}_k^{(i)}}{\sum_k A_{w_k}^{(i)}} \right) \quad (1)$$

where $A_{w_k}^{(i)}$ and $\mathbf{C}_k^{(i)}$ is, respectively, the area and centroid of a patch $w_k^{(i)}$ included in the 1-ring region of the vertex v_i . The function ϕ is used to map the moved vertex onto the target surface. In our method, the vertex v is moved to the control point p closest to the vertex, and ϕ is formulated by

$$\phi(\mathbf{v}) = \frac{\|\mathbf{p}\|}{\|\mathbf{v}\|} \mathbf{v} \quad (2)$$

where $\|y\|$ is the norm of a vector y .

In the process of removing the foldover, all vertices are moved by Eq.(1) and (2). The removal process is repeated until all vertices are not moved.

C. Movement of feature vertices with Free-Form Deformation

By using the original SDM algorithm, feature vertices are mapped to the locations of their corresponding points on the target surface. However, in the foldover removal described in Sec. II-B, all the vertices are moved to achieve foldover-free deformation of the tissue model. After the removal process, the feature vertices may be far from their corresponding points. To correct the positions of the feature vertices, Free-Form Deformation(FFD) [6] is used which deforms a mesh model by deforming the space in which the model is embedded.

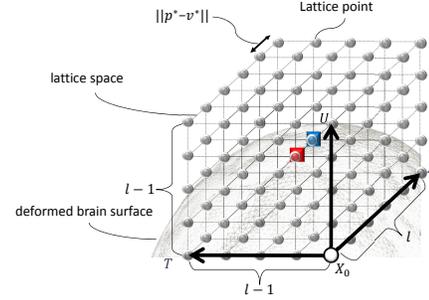


Fig. 1. Movement of feature vertices using FFD. Red and blue points are the feature vertex and its corresponding point.

The use of FFD enables the local surface deformation without foldovers. Owing to this reason, the feature vertices are efficiently moved to their corresponding points by applying FFD with the local area where the feature vertices and their neighbor vertices exist.

To apply FFD, for each feature vertex v^* , 3D control lattice around v^* and its corresponding point p^* is generated as shown in Fig. 1. The local FFD coordinate system is defined by three vectors S, T, U which are orthogonal each other. The axis S is defined by $S = l(\mathbf{p}^* - \mathbf{v}^*)$ (l : an odd number). The axis T is the vector selected from the vectors orthogonal to S . The axis U is obtained by the cross product of S and T . The magnitude of T and U is set to $(l-1)(\mathbf{p}^* - \mathbf{v}^*)$.

To facilitate manipulation of the FFD coordinate system, the axes S, T, U are uniformly divided into a grid by the distance between v^* and p^* . The control lattice comprises l grid points along the S axis, and $l-1$ grid points along the axes T and U (Fig. 1). The global coordinate $K(a, b, c)$ ($0 \leq a \leq l, 0 \leq b, c \leq l-1$) of the (a, b, c) -th grid point is given by

$$\mathbf{K}(a, b, c) = \mathbf{X}_0 + \frac{a}{l} \mathbf{S} + \frac{b}{l-1} \mathbf{T} + \frac{c}{l-1} \mathbf{U} \quad (3)$$

where \mathbf{X}_0 is the origin of the FFD coordinate system, and its location is defined satisfying $\mathbf{v}^* = \mathbf{K}(h, h, h)$ and $\mathbf{p}^* = \mathbf{K}(h+1, h, h)$ ($h = \frac{l-1}{2}$) as follows:

$$\mathbf{X}_0 = \mathbf{v}^* - h \left(\frac{1}{l} \mathbf{S} + \frac{1}{l-1} \mathbf{T} + \frac{1}{l-1} \mathbf{U} \right). \quad (4)$$

The local coordinate (s, t, u) of an arbitrary point \hat{v} in the grid is expressed by

$$\hat{v} = \mathbf{X}_0 + s\mathbf{S} + t\mathbf{T} + u\mathbf{U}. \quad (5)$$

Using the grid points, v^* is moved to coincide with its corresponding point p^* . Since $\mathbf{v}^* = \mathbf{K}(h, h, h)$ and $\mathbf{p}^* = \mathbf{K}(h+1, h, h)$, the movement is made by the FFD deformation using only $\mathbf{K}(h, h, h)$:

$$\mathbf{K}(h, h, h) \leftarrow \mathbf{K}(h, h, h) + \frac{1}{lB} \mathbf{S}; \quad (6)$$

$$B = I_h^l(s_{v^*}) \times I_h^{l-1}(t_{v^*}) \times I_h^{l-1}(u_{v^*}); \quad (7)$$

$$I_n^m(\lambda) = \binom{m}{n} (1-\lambda)^{m-n} \lambda^n \quad (8)$$

where $(s_{v^*}, t_{v^*}, u_{v^*})$ is the location of v^* in the FFD coordinate system. The movement using Eq.(6) causes the local

deformation of the model included in the lattice concurrently. The positions of the vertices in the lattice are updated by

$$\hat{\mathbf{v}} = \sum_a I_a^l(s) \sum_b I_b^{l-1}(t) \sum_c I_c^{l-1}(u) \mathbf{K}(a, b, c). \quad (9)$$

The algorithm for moving the feature vertices is as follows:

- [F1] For each feature vertices,
 - a) Generate the FFD coordinate system and the grid points.
 - b) Move all vertices within the grid by Eqs. (6) and (9).
- [F2] Map the vertices on the target surface by Eq. (2).
- [F3] If all feature vertices is satisfied with $\|\mathbf{p}^* - \mathbf{v}^*\| < \tau$ (τ : threshold), the process is terminated. Otherwise return Step.F1.

III. GEOMETRICAL FEATURE PRESERVING MAPPING

After the movement of feature vertices process, the tissue model is deformed to preserve the original geometric properties as far as possible. This deformation is based on an area- and angle-preserving mapping. The definition of each mapping is described by the example of mapping one triangular mesh model \mathcal{D}^W onto a given parameter domain Ω^* . Here, the mapped model on the domain Ω^* is denoted as \mathcal{D}^* .

An area-preserving mapping is the mapping $\sigma : \mathcal{D}^W \rightarrow \mathcal{D}^*$ if the area of each patch A_w in \mathcal{D}^W is the same as that of the corresponding patch $\sigma(A_w)$ in \mathcal{D}^* . When the total area of \mathcal{D}^W is the same as that of \mathcal{D}^* , the area-preserving mapping is determined by minimizing the distortion metric of the area [7]:

$$\tilde{E}_{area}(\sigma) = \sum_{w \in \mathcal{D}^W} (\sigma(A_w) - A_w)^2. \quad (10)$$

However, in our method, the surface area of the model \mathcal{D}^W is not always equal to that of the mapped model \mathcal{D}^* . Instead of Eq. (10), we define a objective function by

$$E_{area}(\sigma) = \sum_{w \in \mathcal{D}^W} \left| \frac{\sigma(A_w)}{\sum_{w \in \mathcal{D}^W} \sigma(A_w)} - \frac{A_w}{\sum_{w \in \mathcal{D}^W} A_w} \right| \quad (11)$$

$$\approx \sum_i e_{area}(i, \sigma); \quad (12)$$

$$e_{area}(i, \sigma) = \sum_{w_k^{(i)} \in W_i} \left| \frac{\sigma(A_{w_k^{(i)}})}{\sum_{w_k^{(i)} \in W_i} \sigma(A_{w_k^{(i)}})} - \frac{A_{w_k^{(i)}}}{\sum_{w_k^{(i)} \in W_i} A_{w_k^{(i)}}} \right|. \quad (13)$$

An angle-preserving mapping is the mapping $\sigma : \mathcal{D}^W \rightarrow \mathcal{D}^*$ if the angle of each pair of intersecting arcs in \mathcal{D}^W is the same as that of the corresponding arcs in \mathcal{D}^* [7]. In our method, the angle-preserving mapping is determined by minimizing the distortion metric of the angle:

$$E_{angle}(\sigma) = \sum_i e_{angle}(i, \sigma); \quad (14)$$

$$e_{angle}(i, \sigma) = \sum_{w_k^{(i)} \in W_i} \sum_{d=0}^3 |\sigma(\theta_{w_k^{(i)}}^d) - \theta_{w_k^{(i)}}^d|. \quad (15)$$

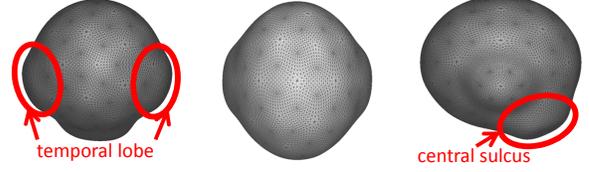


Fig. 2. A simple brain surface:(left: front, middle: top, right: left side)

where θ_w^d is one angle of the patch w .

The distortion metric E is formulated as a linear combination of E_{area} and E_{angle} :

$$E(\sigma) = \mu E_{area} + (1 - \mu) E_{angle} \approx \sum_i e(i, \sigma); \quad (16)$$

$$e(i, \sigma) = \mu \psi e_{area}(i, \sigma) + (1 - \mu) e_{angle}(i, \sigma) \quad (17)$$

where ψ is a scaling factor to adjust the ranges of the two metrics. When a weight coefficient μ ($0 \leq \mu \leq 1$) increases, the property of the distortion metric E change from angle to area-preservation. Therefore, the area- and/or angle-preserving mapping is determined by minimizing the distortion metric in Eq. (16).

Applying a greedy algorithm with Eq.(17), the minimization is the optimization problem of positioning the vertices in the model by moving them repeatedly. Practically, for each vertex \mathbf{v}_i , the suitable next position of \mathbf{v}_i is selected from its candidates $\mathbf{v}_i^{(k)}$ ($k = 1, 2, \dots, N_p^{(i)}$) obtained by

$$\mathbf{v}_i^{(k)} = \mathbf{v}_i + \alpha (\mathbf{u}_k^{(i)} - \mathbf{v}_i) \quad (18)$$

where α ($0 \leq \alpha < 1$) is a coefficient, and $\mathbf{u}_k^{(i)}$ is the vertex connected to \mathbf{v}_i by one edge. To prevent the foldover, the movement of \mathbf{v}_i is limited within its 1-ring region. Among the candidates, \mathbf{v}_i is moved to the candidate with minimum error in Eq.(17).

The algorithm for area- and angle-preserving mapping is as follows:

- [A1] Choose randomly the vertex from all the vertices except the feature vertices.
- [A2] Compute the position candidates of the chosen vertex by using Eq. (18).
- [A3] Move the vertex to the candidate with minimum error in Eq.(17).
- [A4] If all vertices don't moved, go to Step. A1. Otherwise, go to Step. A5.
- [A5] Map the vertices on the target surface by Eq. (2).

IV. EXPERIMENTS

To verify the applicability of our proposed method, we made the experiments using 6 brain surface models and a target surface shown in Fig. 2. The target surface, called a simple brain, has the simplified shape of a human brain. In the experiment, 11 feature vertices are selected from longitudinal fissure of cerebrum, lateral sulcus and central sulcus. The parameters τ in Step.F3 and α in Eq. (18) is set to $\tau = 0.01$ and $\alpha = 0.1$, respectively.

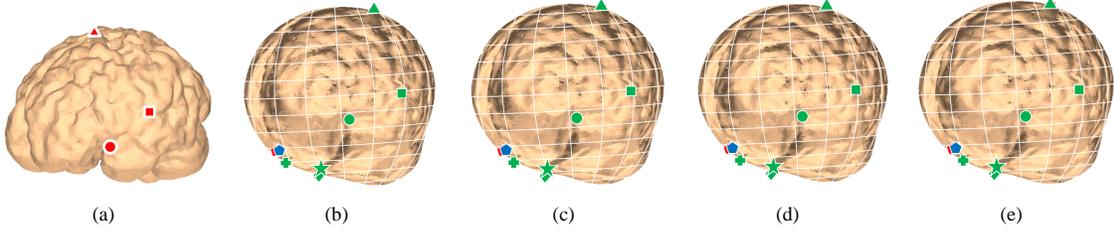


Fig. 3. (a) brain surface model and (b-e) the resulting model by the proposed method: (b) the movement of feature vertices; (c) the area- and (d) angle-preserving mapping; (e) the area- and angle-preserving mapping. Red and blue points are the feature vertices and their corresponding points. Green points denote the feature vertices completely coincided with their corresponding points.

The mapping results of the brain surface model are shown in Fig. 3. Fig. 3(b) show the resulting model by performing the original SDM algorithm and the processes of removing the foldover (Sec. II-B) and moving the feature vertices by FFD (Sec. II-C). All the mapped brain models are completely fitted to the target surface without foldovers, while locating the feature vertices at their target position correctly. Our area- and/or angle-preserving mapping is applied to the model in Fig. 3(b). Fig. 3(c) and Fig. 3(d) show the resulting models by the mapping which preserve area and angle alone, respectively. Fig. 3(e) shows the resulting model obtained by the distortion metric (Eq. 16) with the parameter $\mu = 0.5$. In this case, the mapping leads to the preservation of areas and angles.

To evaluate our area- and/or angle-preserving mapping, we studied the error distributions in areas and angles by the three types of the mappings. The error function ϵ_{area} for area-preservation is formulated by

$$\epsilon_{area} = S \frac{A_w}{\sigma(A_w)} + \frac{1}{S} \frac{\sigma(A_w)}{A_w} - 2; \quad S = \frac{\sum_{w \in W} \sigma(A_w)}{\sum_{w \in W} A_w}. \quad (19)$$

The error function ϵ_{angle} for angle-preservation is calculated by the sum of the absolute difference between the angles of the patches in the mapped model and the original. When the values of the error functions are close to zero, the mappings preserves their corresponding geometric properties. Fig. 4(a) and (b) show the distributions of ϵ_{area} and ϵ_{angle} , respectively. In these figures, three colors correspond to the setting of μ : 0.0(red), 0.5(blue), and 1.0(green). Compared with the distribution (black line in Fig. 4) using the model obtained by FFD deformation Fig. 3(b), our mapping method (red, blue and green lines) reduces the errors on areas and angles. In the case of $\mu = 1$ (green line), our mapping is the area-preserving mapping, and the distribution of ϵ_{area} has the highest peak at zero. The distribution of ϵ_{angle} becomes narrow with decreasing the value of μ . For $\mu = 0$ (red line), our mapping is the angle-preserving mapping. In this case, the distribution of ϵ_{angle} is steep, and has the single peak at zero. From these results, our method can achieve the preservation of the geometrical features.

V. CONCLUSION

In this paper, we proposed the method of mapping of the brain surface model with complex shape onto an arbitrary target surface. The model is mapped roughly on the target surface by SDM method. After removing the foldover, the local surfaces including the feature vertices is deformed by FFD to move the feature vertices to their corresponding points without

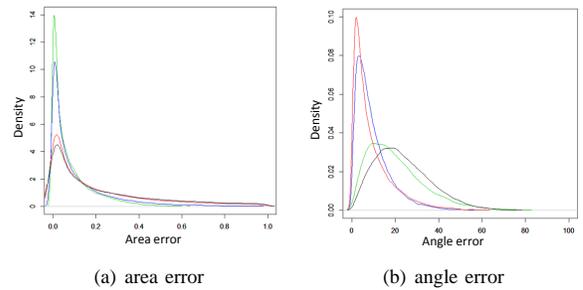


Fig. 4. Error distributions in areas and angles by the proposed mapping method. black: without geometrical process, red: $\mu = 0$, blue: $\mu = 0.5$, green: $\mu = 1$.

foldovers. Moreover, the model is deformed to preserve area and angle before and after mapping. From the experimental results, our method can map a brain model on a target surface while both controlling feature vertices position and keeping geometrical features without foldovers.

ACKNOWLEDGMENT

This work was supported by JSPS KAKENHI Grant Number 23500244, 24390345, 24103708 and the Ministry of Health, Labour and Welfare(201024171A).

REFERENCES

- [1] H. Kobatake, "Future cad in multi-dimensional medical images:—project on multi-organ, multi-disease cad system—," *Computerized Medical Imaging and Graphics*, vol. 31, no. 4, pp. 258–266, 2007.
- [2] T. Heimann, H.-P. Meinzer *et al.*, "Statistical shape models for 3d medical image segmentation: A review," *Medical image analysis*, vol. 13, no. 4, p. 543, 2009.
- [3] K. Morooka and H. Nagahashi, "Self-organizing deformable model : a method for projecting a 3d object mesh model onto a target surface," in *InTech Computer Graphics*, 2012, pp. 19–38.
- [4] S. Matsui, K. Aoki, and H. Nagahashi, "3d triangular mesh parameterization with semantic features based on competitive learning methods," *IEICE TRANSACTIONS on Information and Systems*, vol. 91, no. 11, pp. 2718–2726, 2008.
- [5] T. Athanasiadis, I. Fudos, C. Nikou, and V. Stamati, "Feature-based 3d morphing based on geometrically constrained sphere mapping optimization," in *Proceedings of the 2010 ACM Symposium on Applied Computing*. ACM, 2010, pp. 1258–1265.
- [6] T. Sederberg, "Free-from deformation of solid geometric models," in *Proceedings of ACM SIGGRAPH in Computers and Graphics*, 20(4), 1986, pp. 151–160.
- [7] M. S. Floater and K. Hormann, "Surface parameterization: a tutorial and survey," in *Advances in multiresolution for geometric modelling*. Springer, 2005, pp. 157–186.